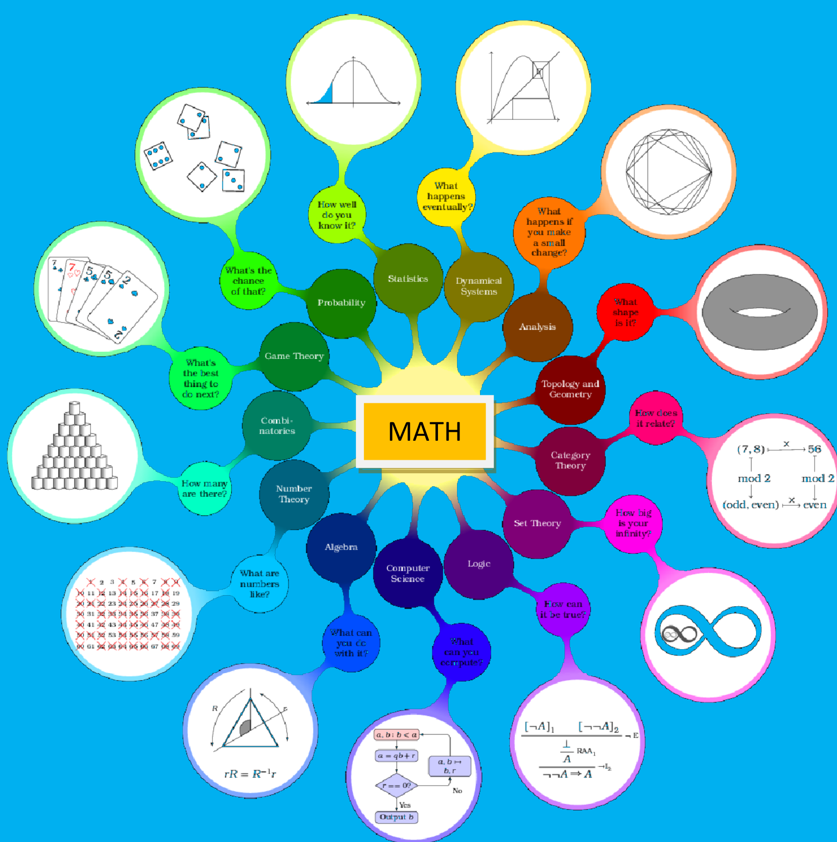
e-Bridging Module

FES2021@MATAT455



Authors

Nor Hanim Abd Rahman, Siti Nurleena Abu Mansor
Ahmad Rashidi Azudin, Siti Mariam Saad
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Fuziatul Norsyiha Ahmad Shukri
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Published and edited by the Unit Penulisan dan Penerbitan JSKM,
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Preface

This modular workbook entitled "***e-Bridging Module: FES2021@ MAT455***" is designed as a revision and preparation for the students who are taking Calculus subjects especially during their first semester undergraduate. Thus, it is presented in a simple language, but in structured exercises manner.

This module is divided into four chapters, which are Common Math Errors, Exploring with Calculator, Geogebra (2D & 3D), and Matrix & Vectors. It is filled with information to give a ready reference and instructional material aims to help students to understanding and recognize the fundamental operations that they are going to use in their syllabus and it also introduces some skills for them to solve the mathematical problems more effectively and to avoid doing the unnecessarily mistakes.

The authors believe that after finishing this modular workbook, students will be able to feel more confident, and perform better in their studies.

Study Smart & Happy Learning!

~ DrNhar

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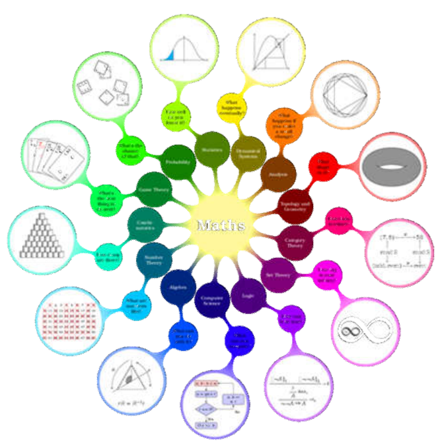
Module 1

Common Math Errors

Presented by

Nor Hanim Abd Rahman

Siti Nurleena Abu Mansor



MODULE 1

COMMON ERRORS IN MATH

PREPARED BY
NOR HANIM ABD RAHMAN & SITI NURLEENA ABU MANSOR

INTRODUCTION

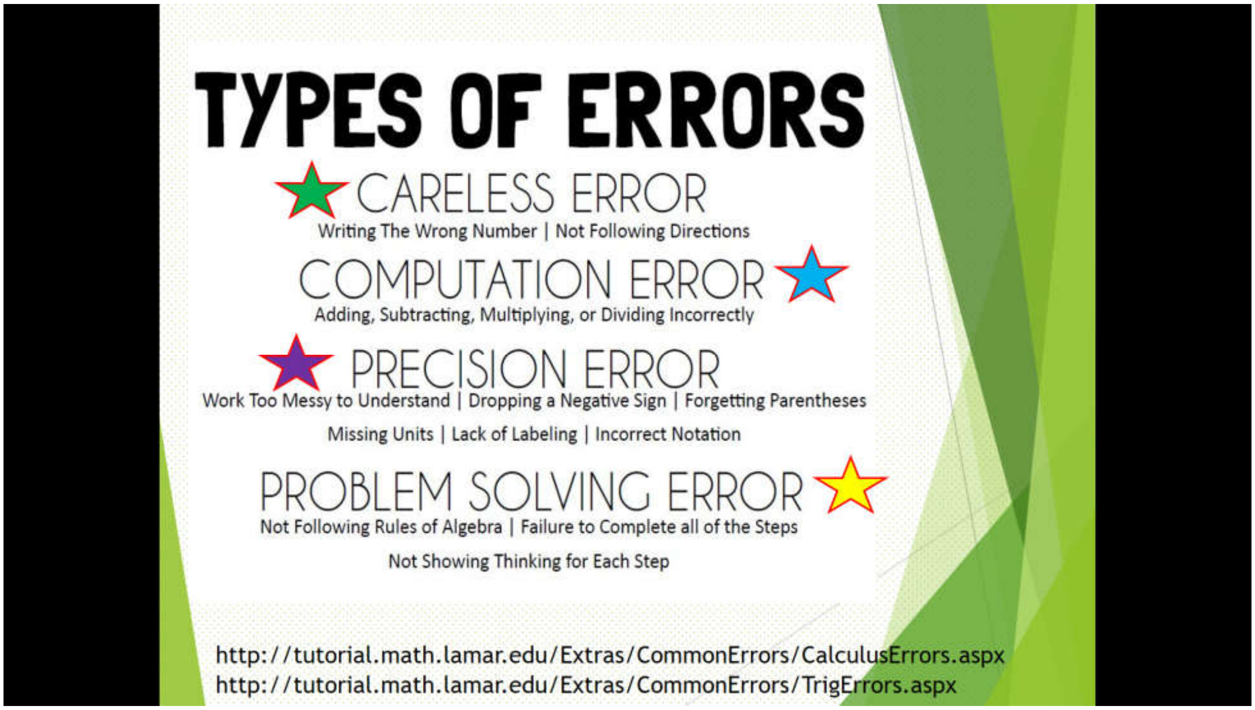
When you are learning a new skill, it requires practice, and making mistakes is part of the process. Making mistakes is common and it is a global problems. However, making mistakes in math is a good thing! It is part of the process in learning and understanding more deeply; understanding how to prevent them and how to learn from them is essential. There are different types of math errors that students may make and encounter such as careless errors, computational errors and conceptual errors.
In this module, students will learn how to recognize, classify and overcome the problems.

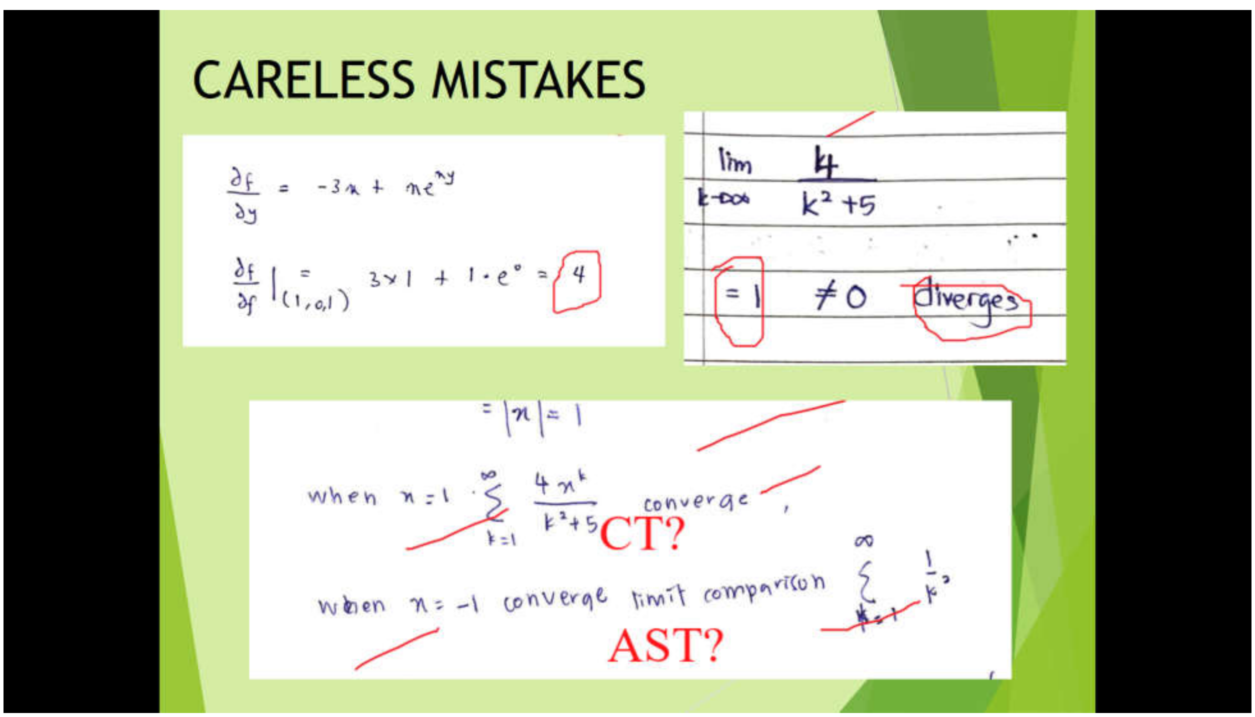
BENEFITS OF ANALYZING ERRORS:

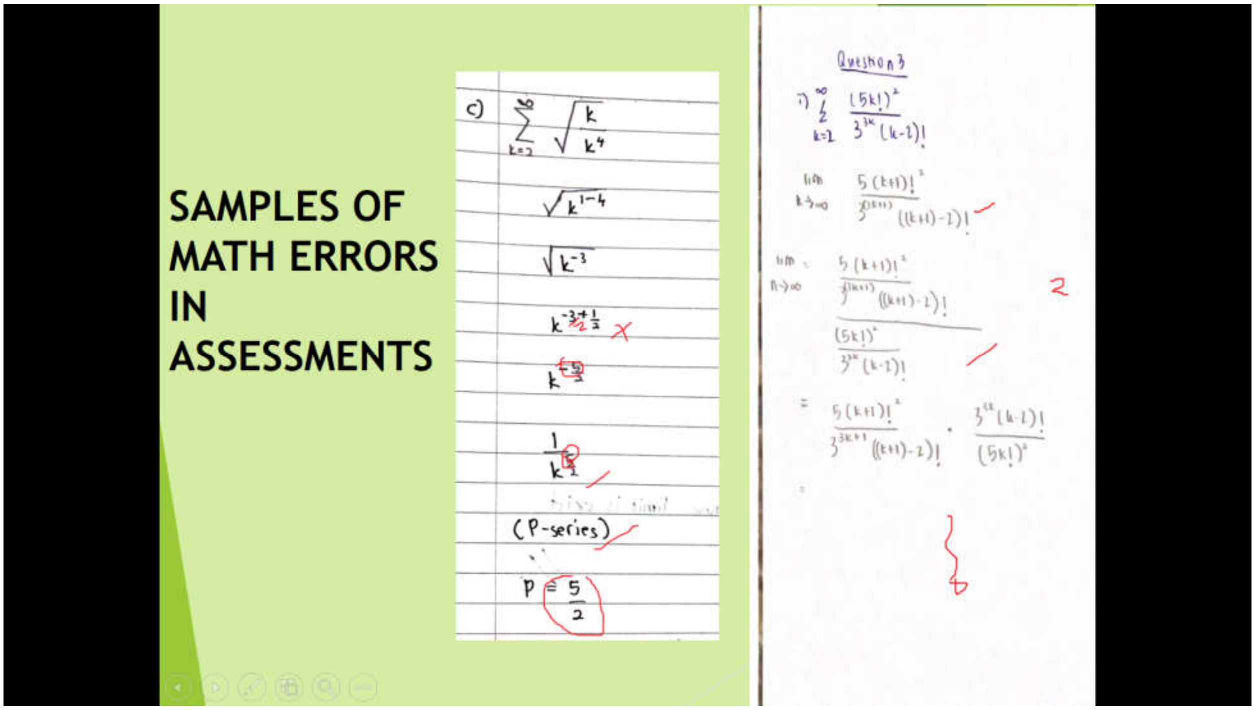
- Learn from own mistakes and carelessness.
- It will take some time to recognize and classify the mistakes but once you do, you will see the process of solving the problems clearer.

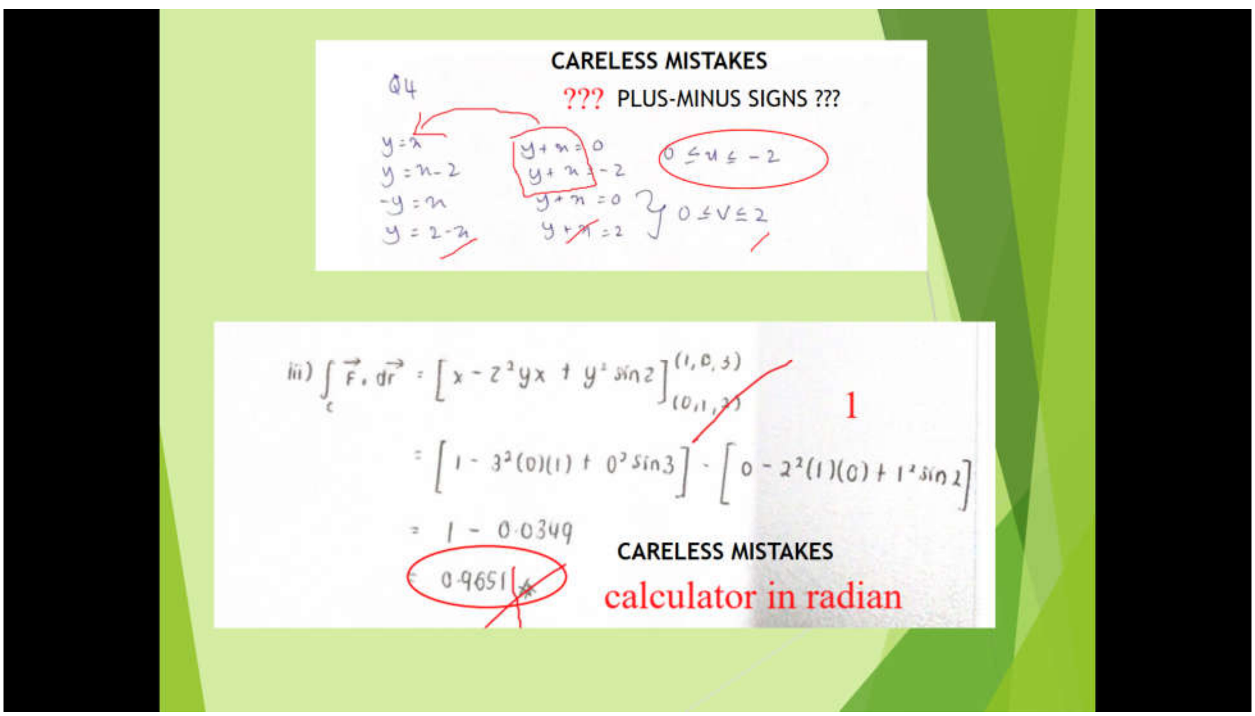
COMMON ERRORS DONE IN MATHEMATIS & SAMPLES

ERROR 1:

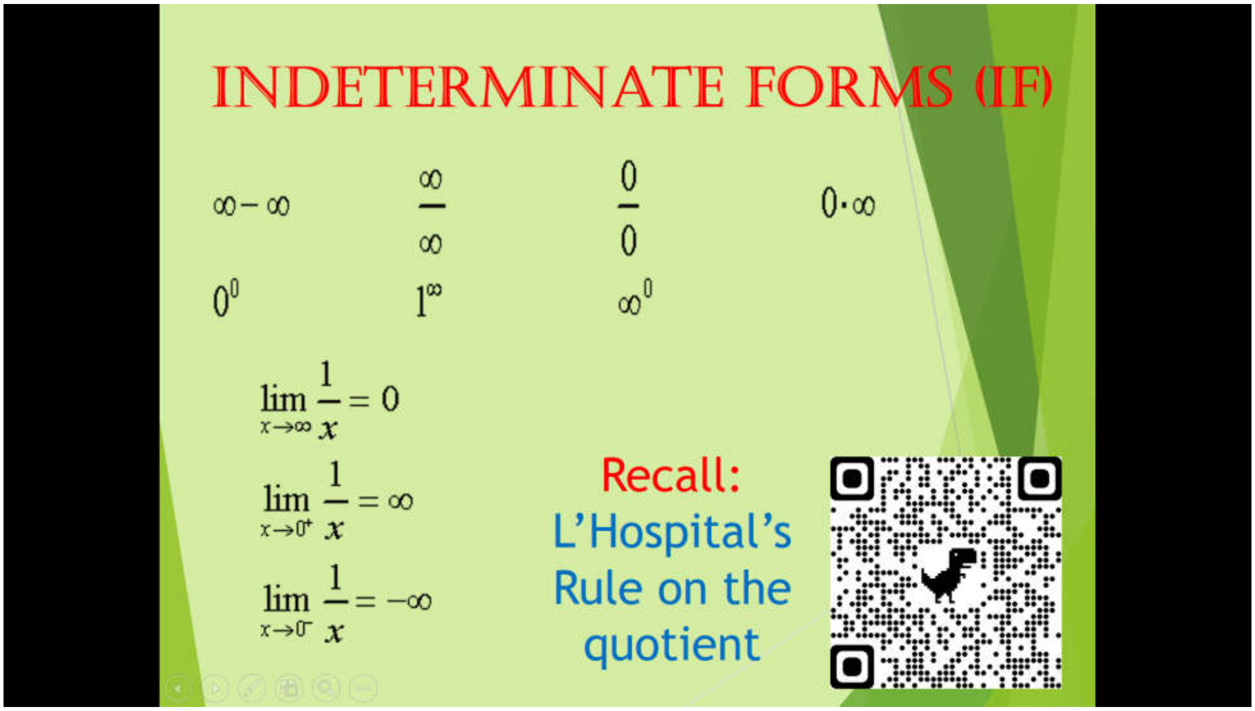


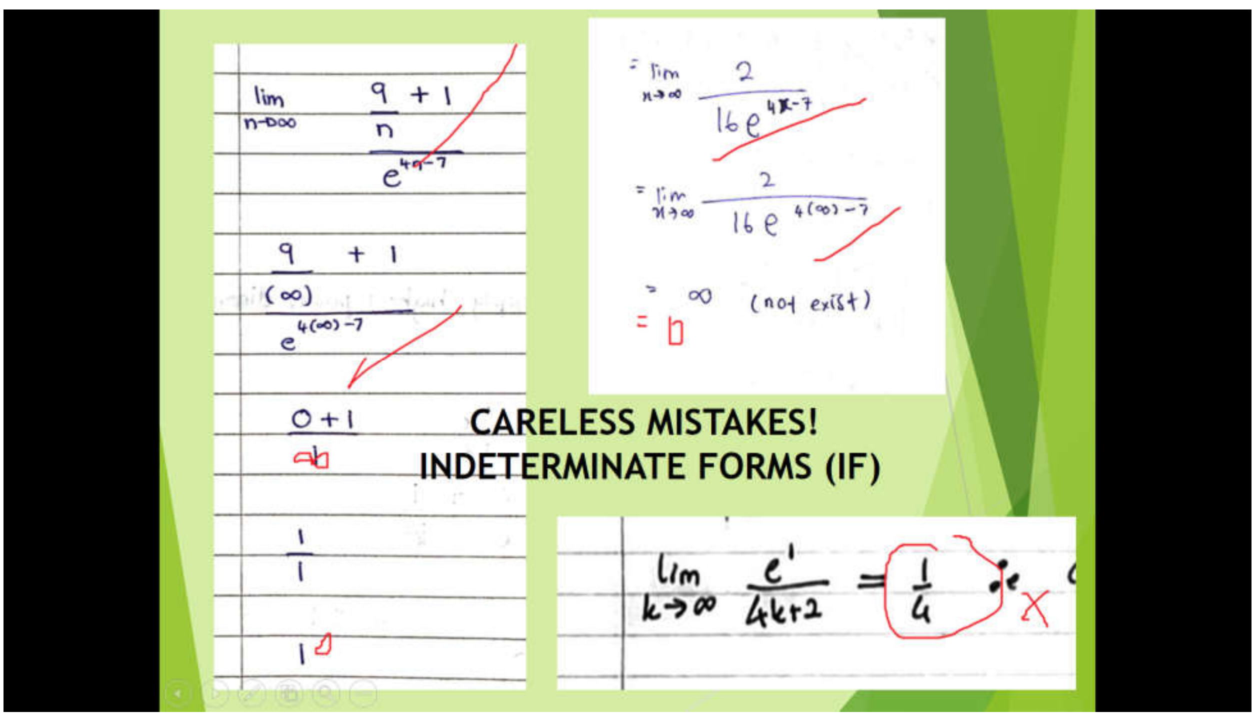



SAMPLES OF
MATH ERRORS
IN
ASSESSMENTS
Question 3
(P-series)

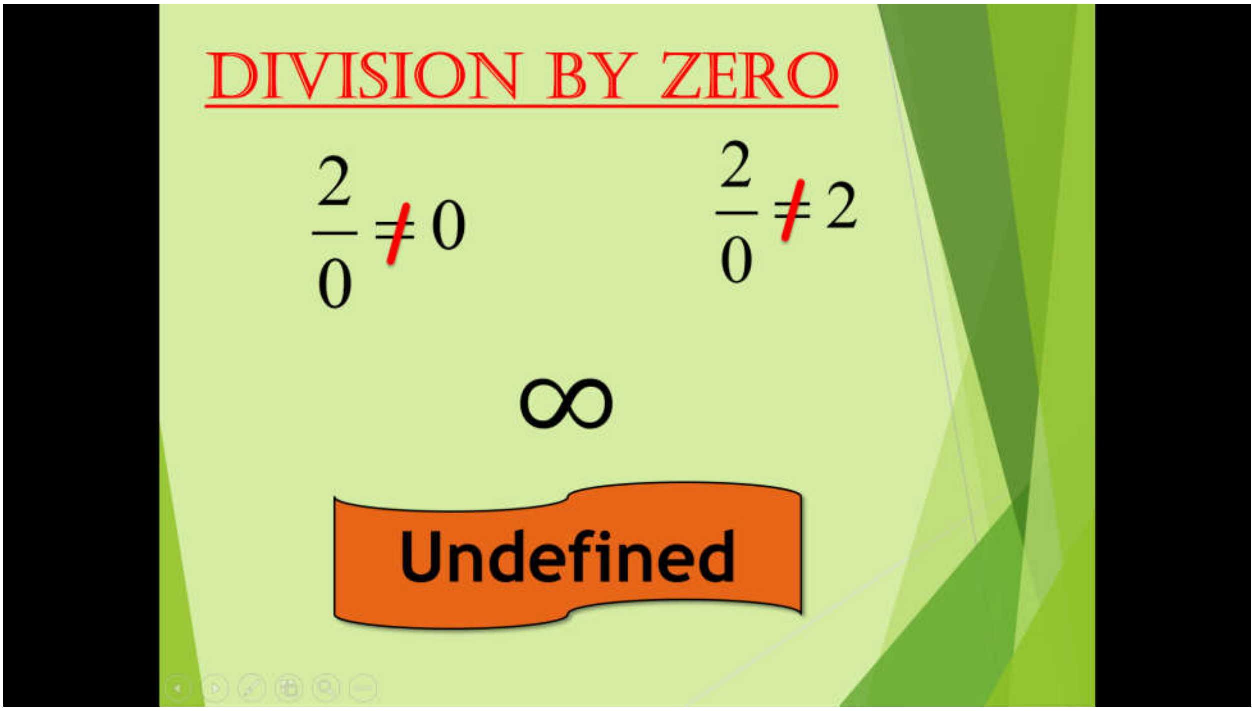

CARELESS MISTAKES
??? PLUS-MINUS SIGNS ???
Q4
CARELESS MISTAKES
calculator in radian

ERROR 2:

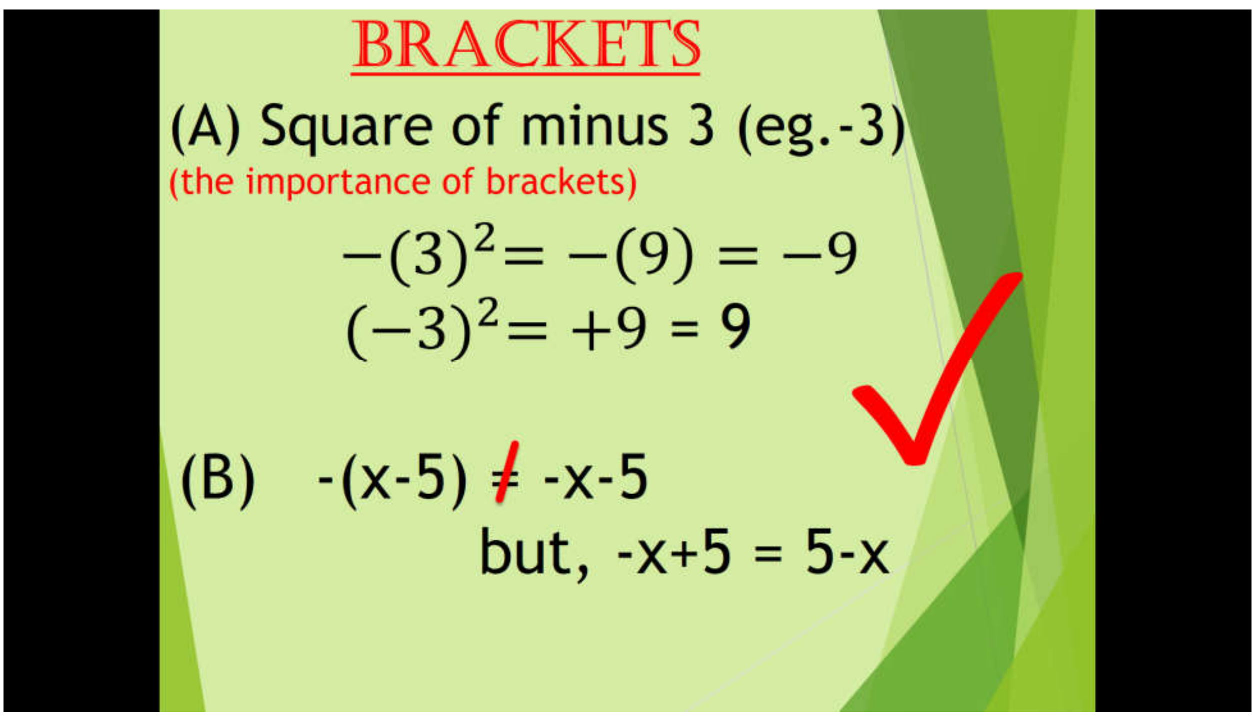

INDETERMINATE FORMS (IF)
$\infty - \infty$ $\frac{\infty}{\infty}$ $\frac{0}{0}$ $0 \cdot \infty$
0^0 1^∞ ∞^0
$\lim_{x\to\infty} \frac{1}{x} = 0$
$\lim_{x\to 0^+} \frac{1}{x} = \infty$
$\lim_{x\to 0^-} \frac{1}{x} = -\infty$
Recall:
L'Hospital's Rule on the quotient

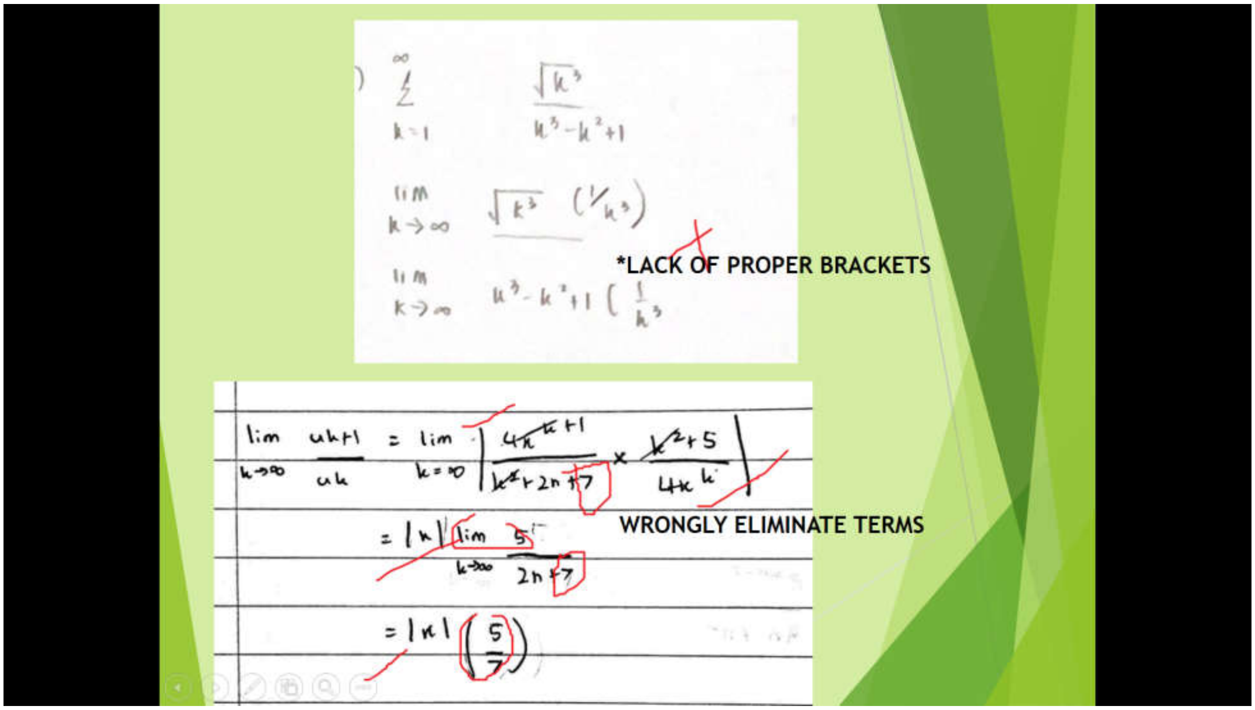

CARELESS MISTAKES!
INDETERMINATE FORMS (IF)
(not exist)

ERROR 3:

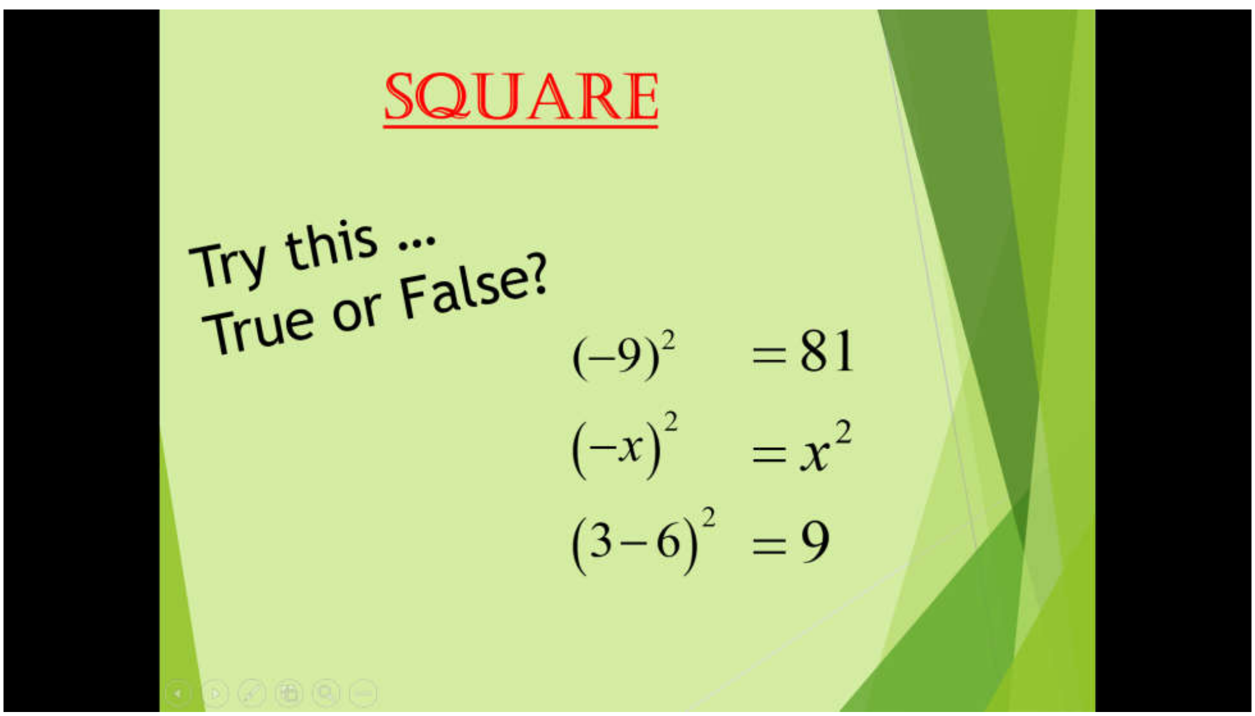


ERROR 4:

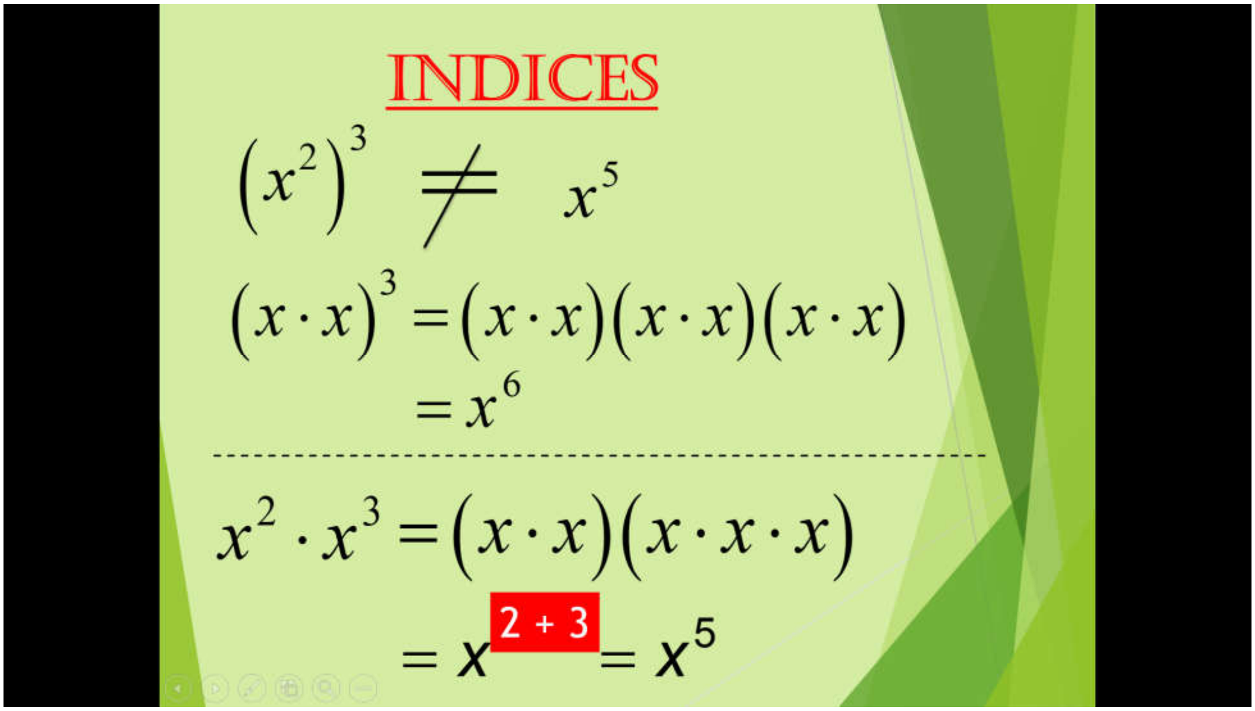


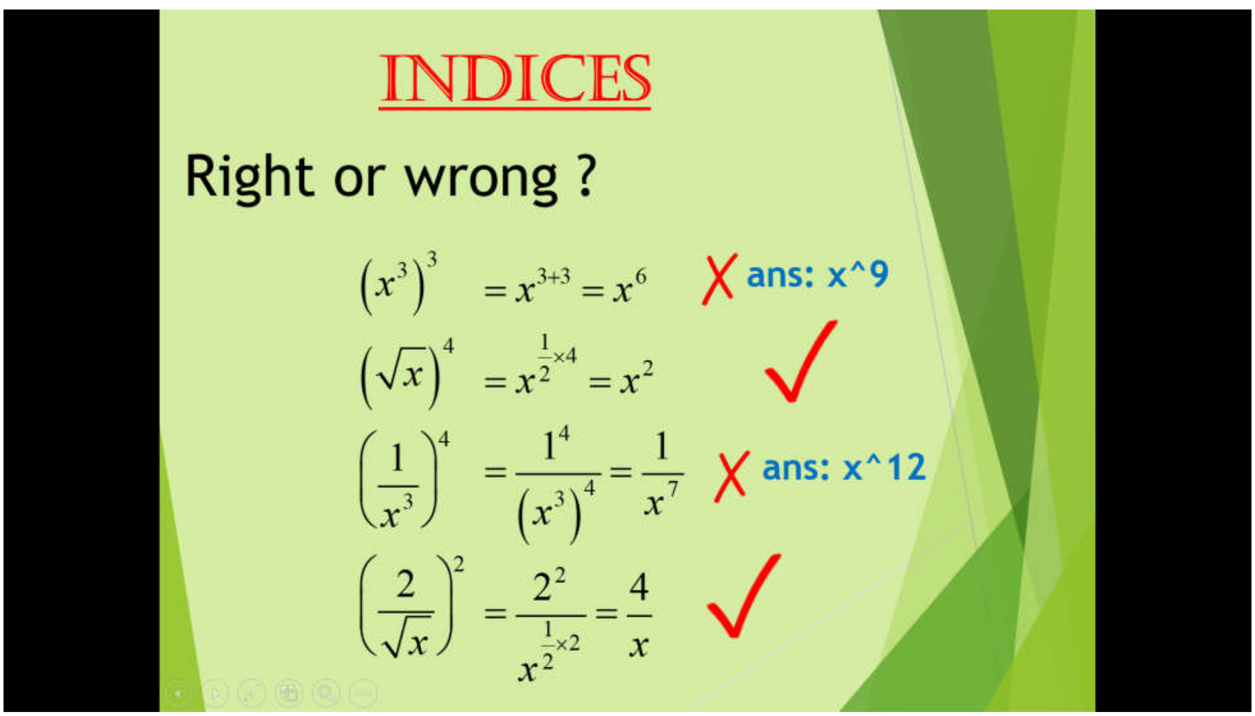

*LACK OF PROPER BRACKETS
WRONGLY ELIMINATE TERMS

ERROR 5:

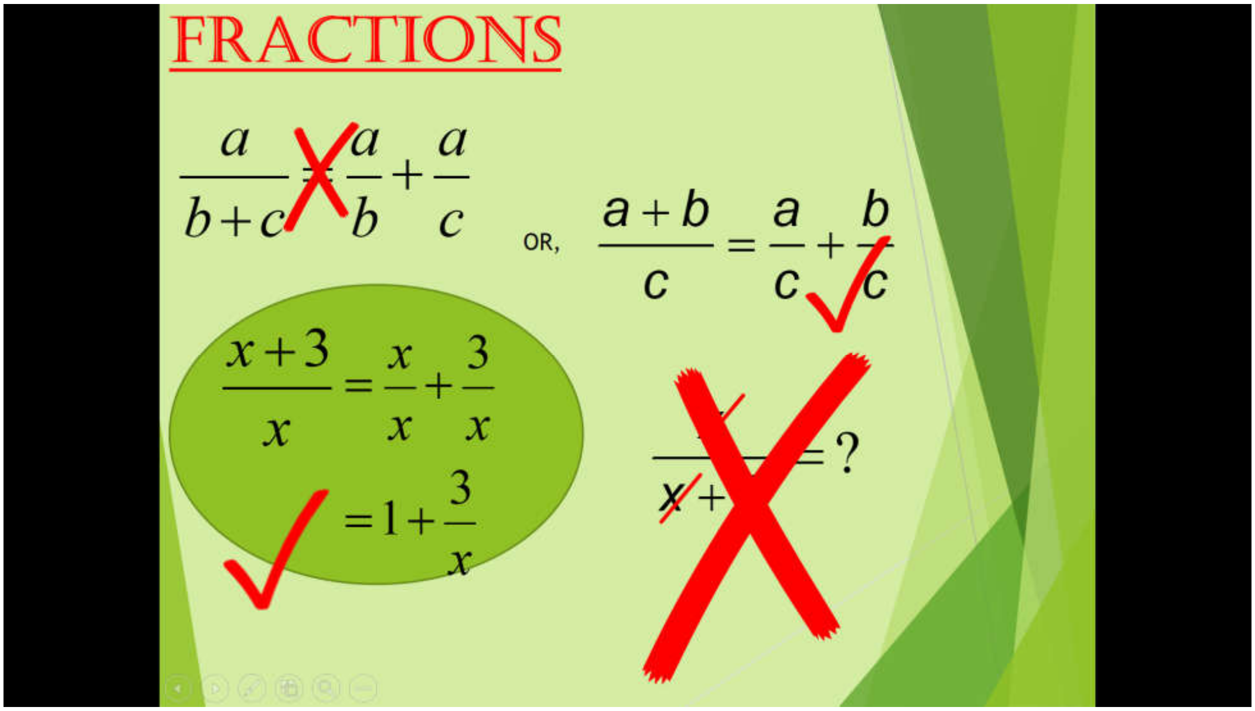

SQUARE
Try this ...
True or False?
$(-9)^2 = 81$
$(-x)^2 = x^2$
$(3-6)^2 = 9$

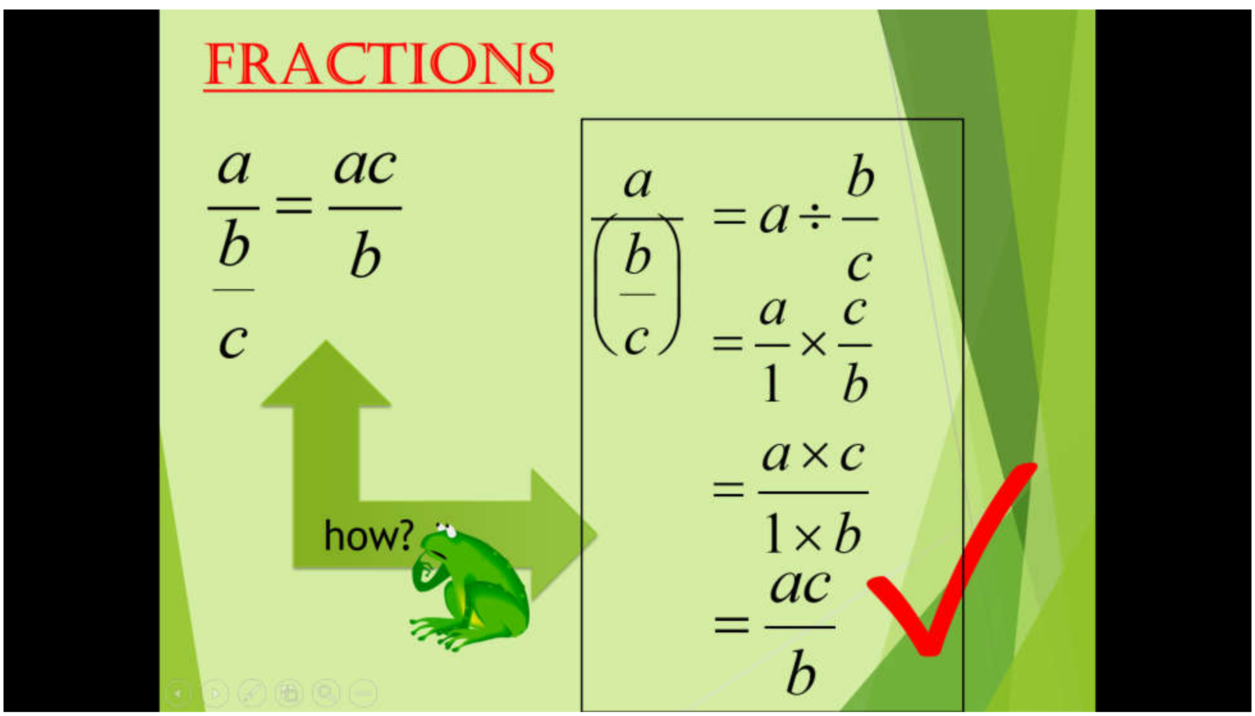
ERROR 6:


INDICES
$\left(x^{2}\right)^{3} \neq x^{5}$
$(x \cdot x)^{3} = (x \cdot x)(x \cdot x)(x \cdot x)$
$= x^{6}$
$x^{2} \cdot x^{3} = (x \cdot x)(x \cdot x \cdot x)$
$= x^{2+3} = x^{5}$

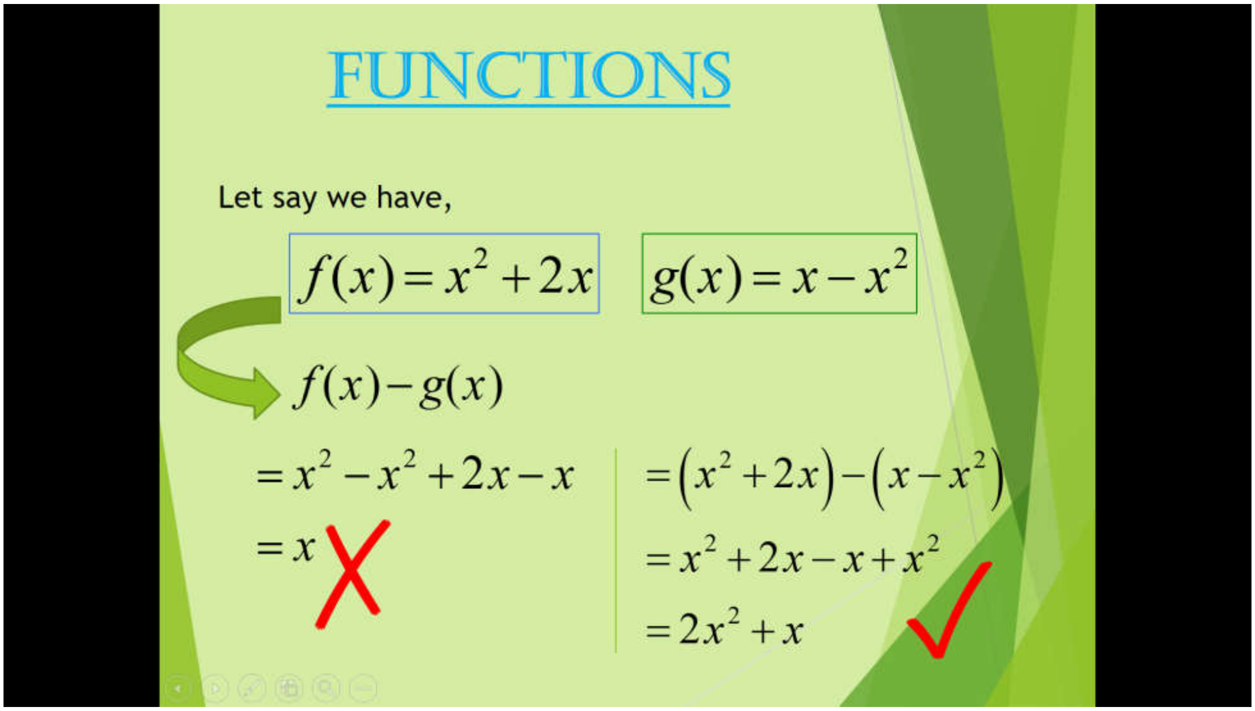

INDICES
Right or wrong ?
$\left(x^{3}\right)^{3} = x^{3+3} = x^{6}$
ans: x^9
$\left(\sqrt{x}\right)^{4} = x^{\frac{1}{2}\times 4} = x^{2}$
$\left(\frac{1}{x^{3}}\right)^{4} = \frac{1^{4}}{\left(x^{3}\right)^{4}} = \frac{1}{x^{7}}$
ans: x^12
$\left(\frac{2}{\sqrt{x}}\right)^{2} = \frac{2^{2}}{x^{\frac{1}{2}\times 2}} = \frac{4}{x}$

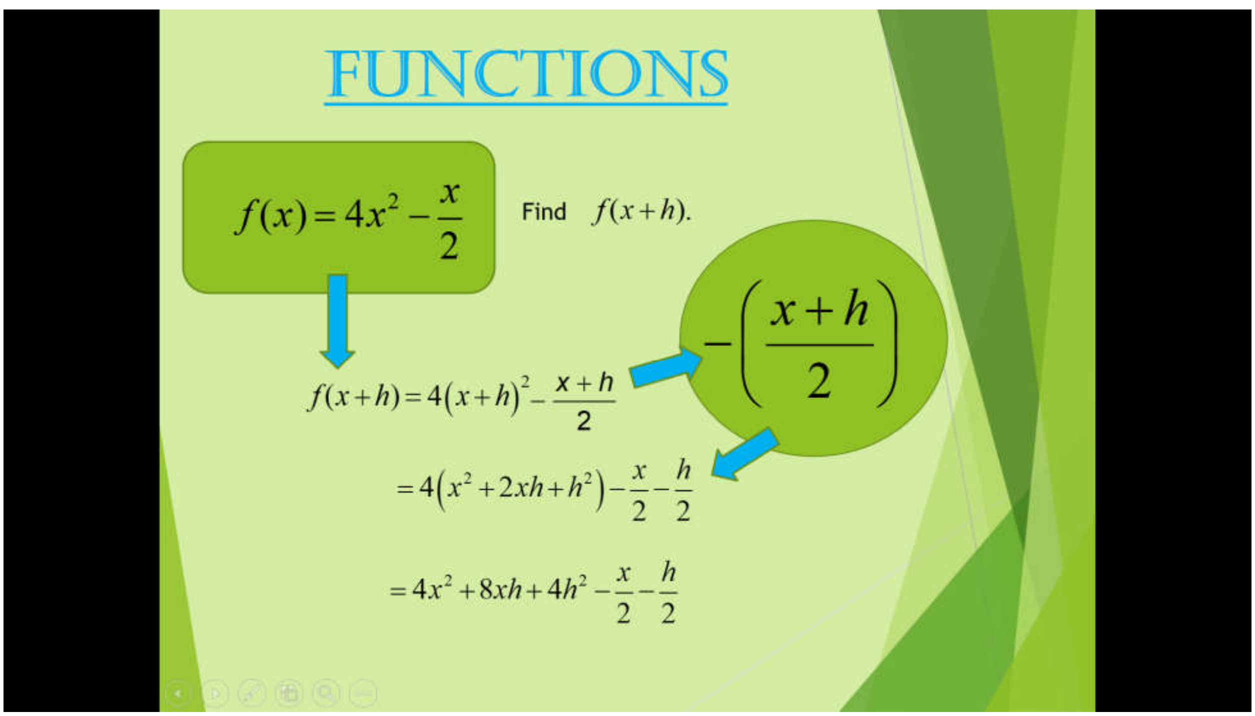
ERROR 7:


FRACTIONS
$\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$
OR,
$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$
$\frac{x+3}{x} = \frac{x}{x} + \frac{3}{x}$
$= 1 + \frac{3}{x}$
= ?

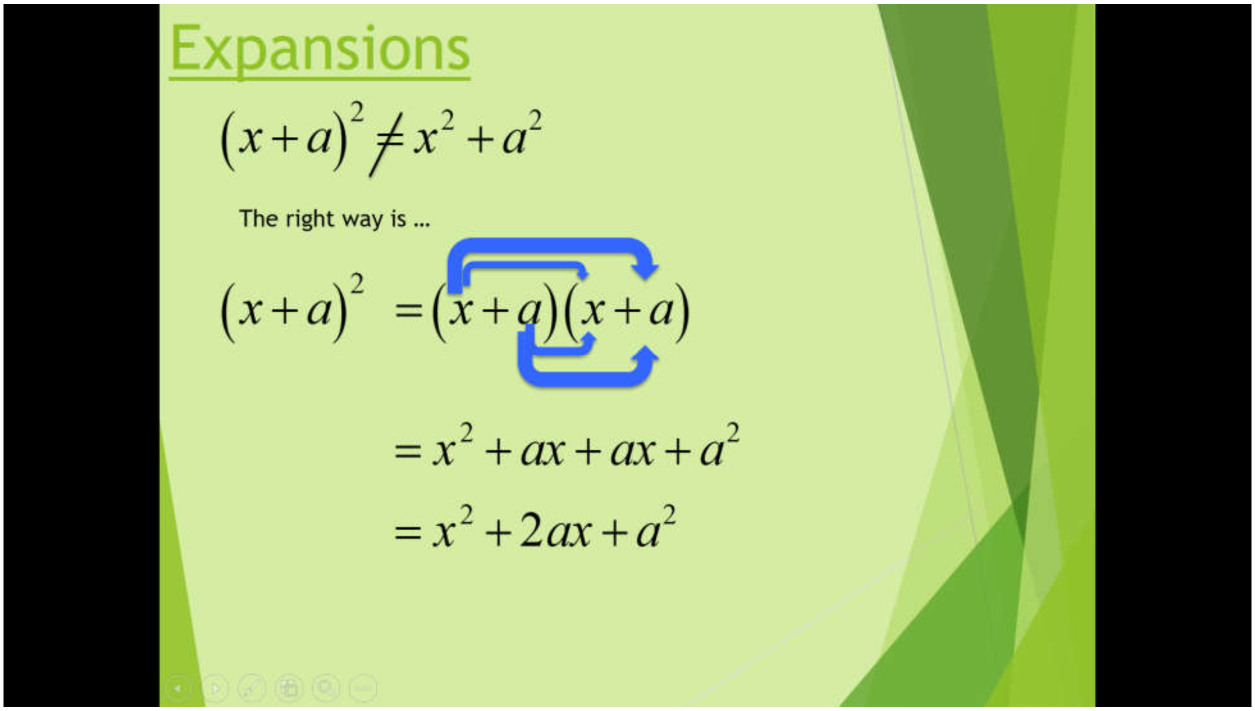

FRACTIONS
$\frac{\frac{a}{b}}{c} = \frac{ac}{b}$
how?
$\frac{a}{\left(\frac{b}{c}\right)} = a \div \frac{b}{c}$
$= \frac{a}{1} \times \frac{c}{b}$
$= \frac{a \times c}{1 \times b}$
$= \frac{ac}{b}$

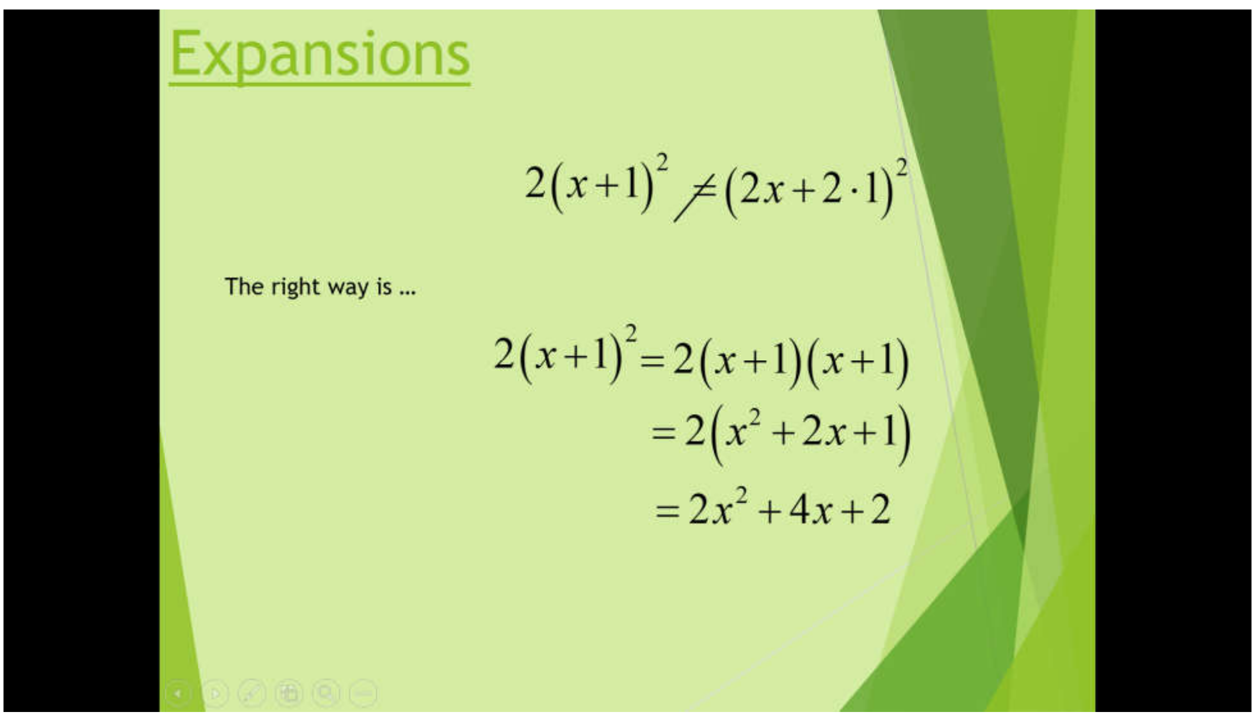
ERROR 8:


FUNCTIONS
Let say we have,
$f(x)=x^2+2x$
$g(x)=x-x^2$
$f(x)-g(x)$
$=x^2-x^2+2x-x$
$=x$
$=(x^2+2x)-(x-x^2)$
$=x^2+2x-x+x^2$
$=2x^2+x$

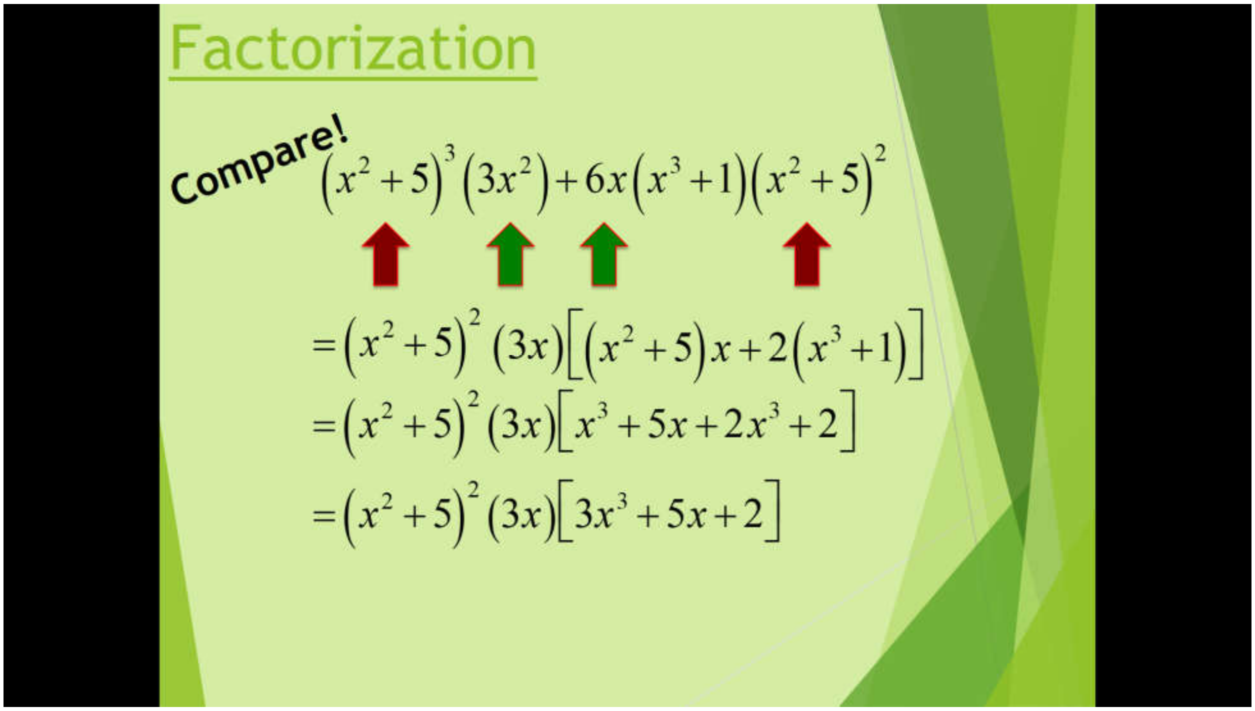

FUNCTIONS
$f(x)=4x^2-\frac{x}{2}$
Find $f(x+h)$.
$-\left(\frac{x+h}{2}\right)$
$f(x+h)=4(x+h)^2-\frac{x+h}{2}$
$=4(x^2+2xh+h^2)-\frac{x}{2}-\frac{h}{2}$
$=4x^2+8xh+4h^2-\frac{x}{2}-\frac{h}{2}$

ERROR 9:

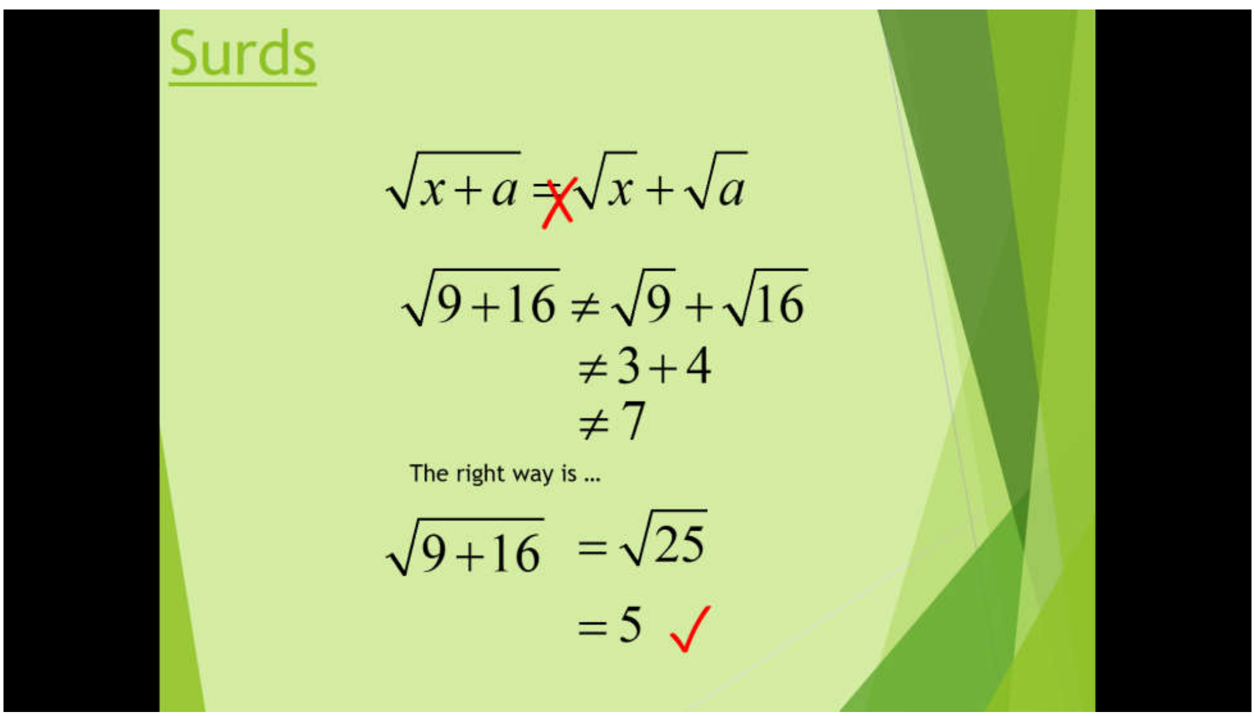

Expansions
$(x+a)^2 \neq x^2 + a^2$
The right way is ...
$(x+a)^2 = (x+a)(x+a)$
$= x^2 + ax + ax + a^2$
$= x^2 + 2ax + a^2$


Expansions
$2(x+1)^2 \neq (2x+2\cdot 1)^2$
The right way is ...
$2(x+1)^2 = 2(x+1)(x+1)$
$= 2(x^2+2x+1)$
$= 2x^2+4x+2$

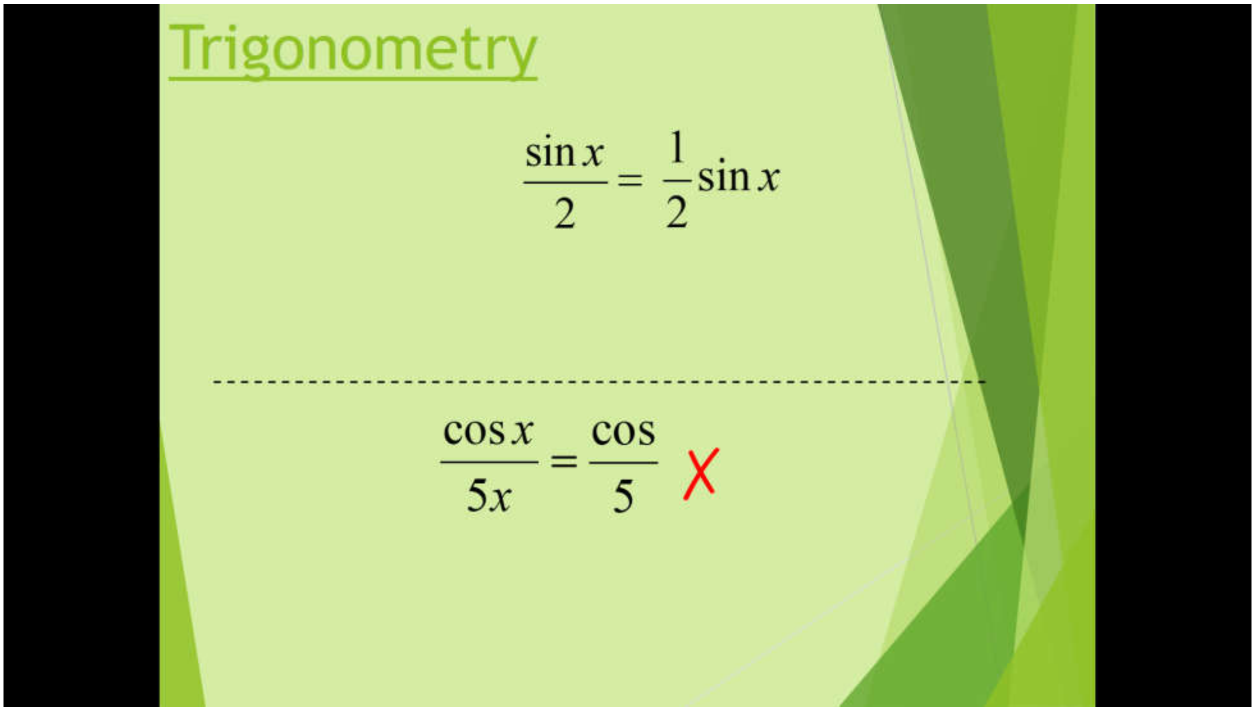
ERROR 10:

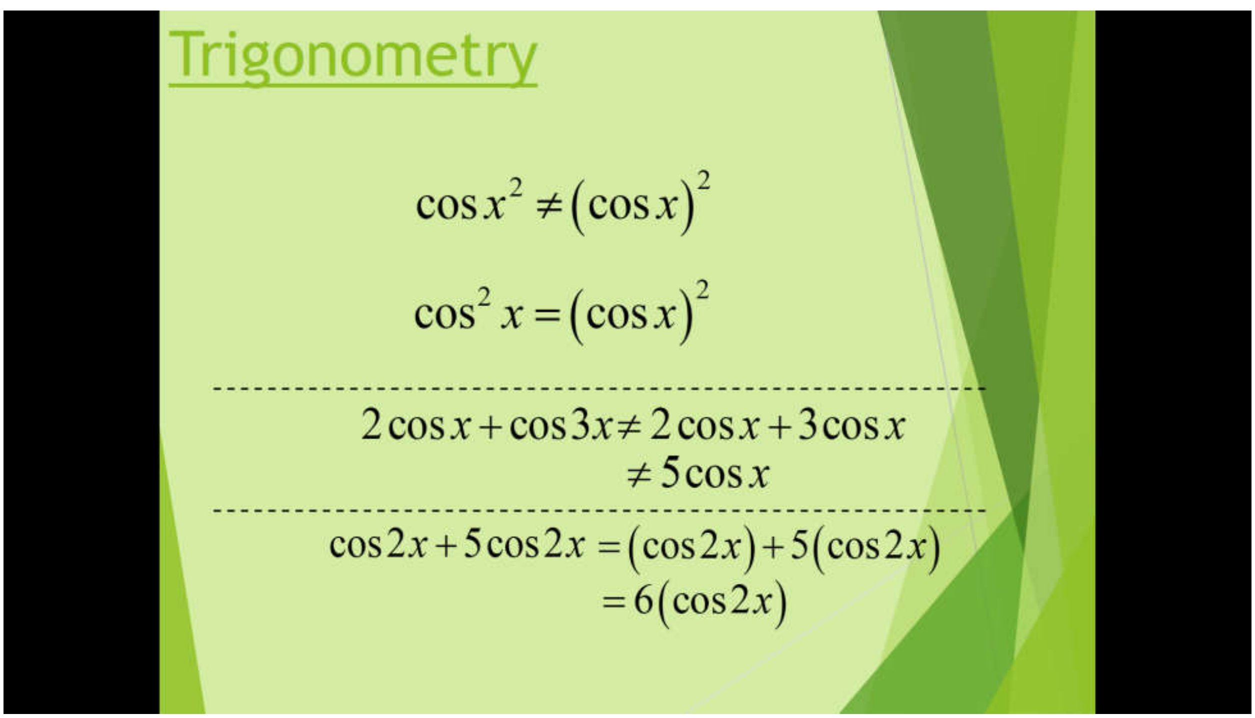


ERROR 11:

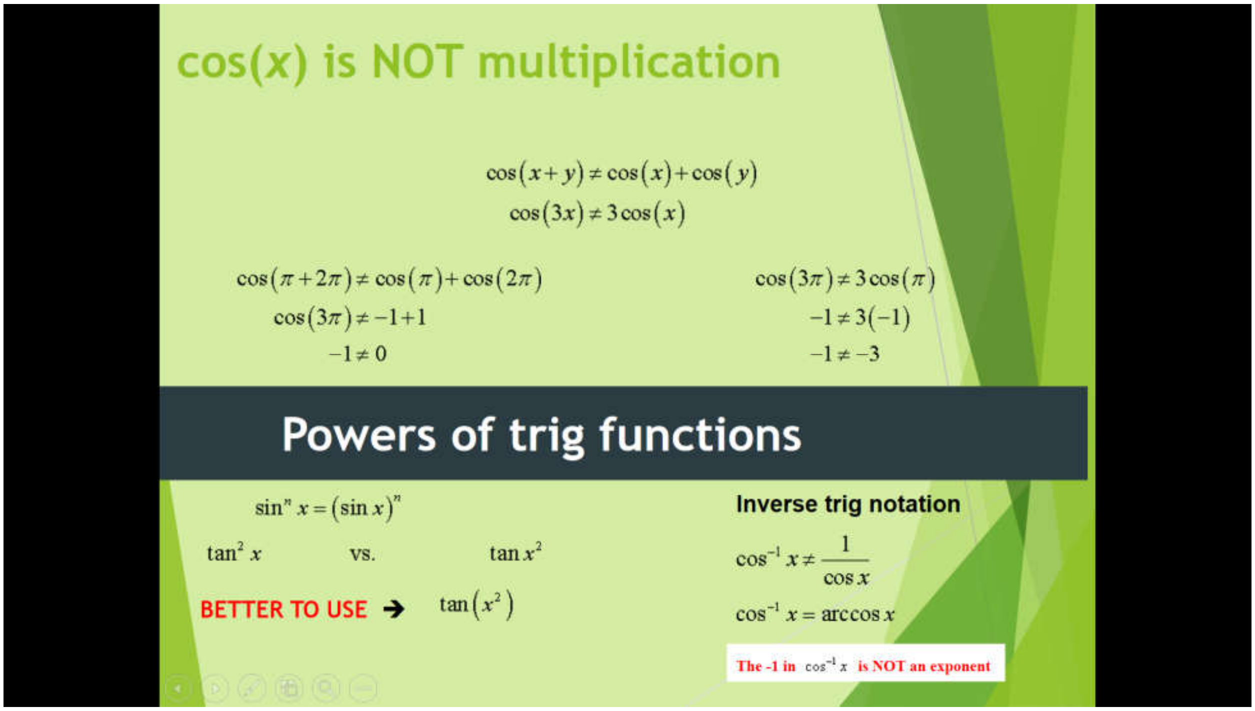


ERROR 12:


Trigonometry
$\frac{\sin x}{2} = \frac{1}{2}\sin x$
$\frac{\cos x}{5x} = \frac{\cos}{5}$


Trigonometry
$\cos x^2 \neq (\cos x)^2$
$\cos^2 x = (\cos x)^2$
$2\cos x + \cos 3x \neq 2\cos x + 3\cos x$
$\neq 5\cos x$
$\cos 2x + 5\cos 2x = (\cos 2x) + 5(\cos 2x)$
$= 6(\cos 2x)$

ERROR 13:


cos(x) is NOT multiplication
$\cos(x+y) \neq \cos(x)+\cos(y)$
$\cos(3x) \neq 3\cos(x)$
$\cos(\pi+2\pi) \neq \cos(\pi)+\cos(2\pi)$
$\cos(3\pi) \neq -1+1$
$-1 \neq 0$
$\cos(3\pi) \neq 3\cos(\pi)$
$-1 \neq 3(-1)$
$-1 \neq -3$
Powers of trig functions
$\sin^n x = (\sin x)^n$
$\tan^2 x$ vs. $\tan x^2$
BETTER TO USE ➔ $\tan(x^2)$
Inverse trig notation
$\cos^{-1} x \neq \frac{1}{\cos x}$
$\cos^{-1} x = \arccos x$
The -1 in $\cos^{-1} x$ is NOT an exponent

EXERCISE

Find the error with the following answers, then give the correct answer:

Question 1

1. $(x + y)^2 = x^2 + y^2$
2. $\dfrac{1}{x+y} = \dfrac{1}{x} + \dfrac{1}{y}$
3. $\sin(x + y) = \sin x + \sin y$

Question 2

1. $\ln(x + y) = \ln x + \ln y$
2. $\sin x^2 = \sin^2 x$
3. $3(2)^x = 6^x$

Question 3

1. $x(y+z) = xy + z$
2. $\cos 4x = 4\cos x$
3. $x - (x + 3) = x - x + 3$

Question 4

1. $x - (x - 3) = x - x - 3$
2. $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$
3. $\sin^{-1} x = \dfrac{1}{\sin x}$

FES455

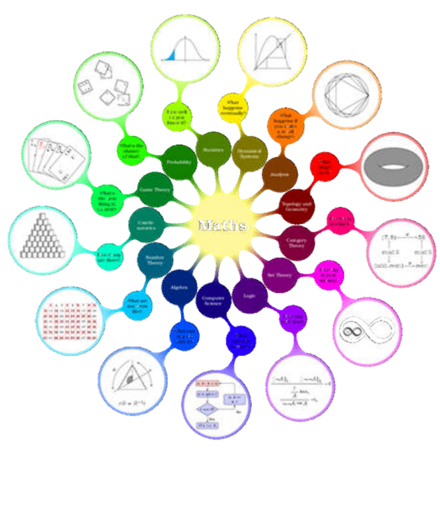
Module 2

Exploring with Calculator

Presented by

Ahmad Rashidi Azudin

Siti Mariam Saad



MODULE 2

SCIENTIFIC CALCULATOR FX-570MS:

1. DIFFERENTIATION AND INTEGRATION
2. FACTORING A SIMPLE QUADRATIC AND SOLVING A SYSTEM OF LINEAR EQUATIONS

PREPARED BY

AHMAD RASHIDI AZUDIN & SITI MARIAM BINTI SAAD

INTRODUCTION

The FX-570MS is one of the most popular scientific calculators used by students and teachers in Malaysia. It is a powerful calculation device which has a simple and easy to use design. It has many features with high functionality and excellent performance as well as good operability.

In this module, students will learn how to use a scientific calculator FX-570MS to:

1. evaluate derivatives of functions at specific values and solve definite integrals
2. factor simple quadratic functions and solve systems of linear equations

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BENEFITS:

- Promotes accuracy in solving mathematics problems.
- Makes learning mathematics more enjoyable and meaningful.

GETTING STARTED WITH EVALUATING THE DERIVATIVE OF FUNCTIONS AT A SPECIFIC VALUE

There are two inputs required to compute the derivative of functions at a specific value using the feature provided by the 570MS calculator:

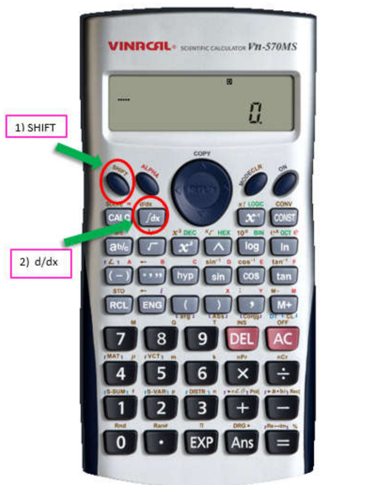
a) a function of variable *x*

b) the value of *x* at which the derivative is calculated

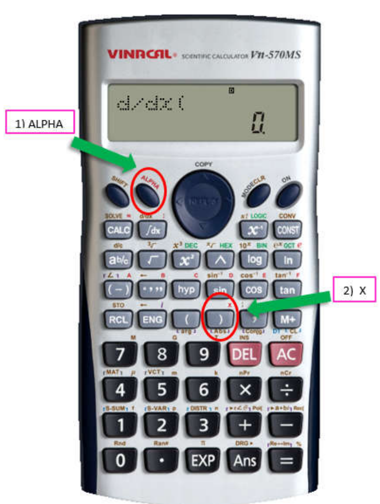
$$\frac{d}{dx}[\text{a function}]\Big|_{x\,=\,\text{the specific value}}$$

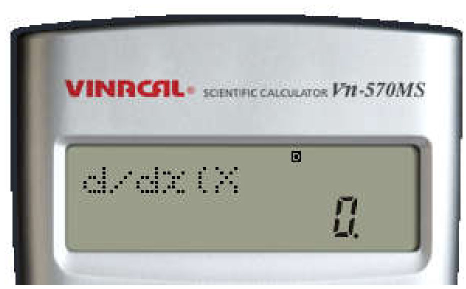
Format: $\boxed{\textbf{SHIFT}}$ $\boxed{\text{d/dx}}$ *a function* $\boxed{,}$ *the value* $\boxed{)}$ $\boxed{=}$

STEP 1 : Press "**SHIFT**", then "**d / dx** " to activate the differential operator.

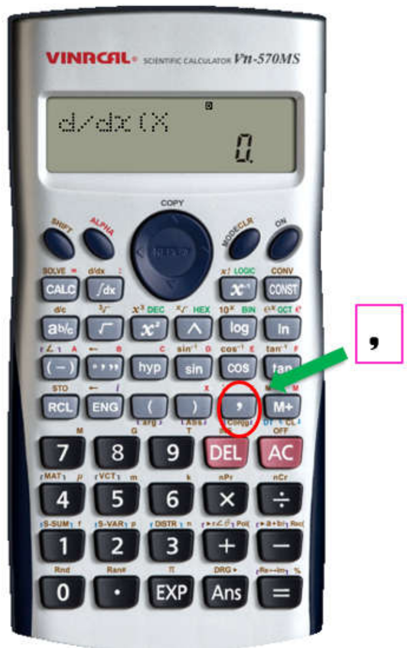


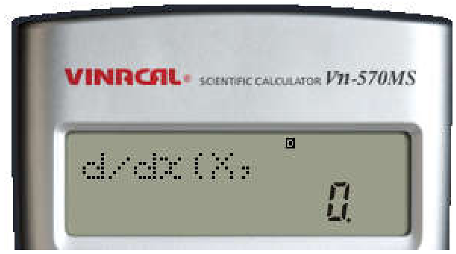
STEP 2 : Enter a function of *x*. If the function consists of the variable *x*, we may need to press "**ALPHA**", then "**)**" to key in the variable.



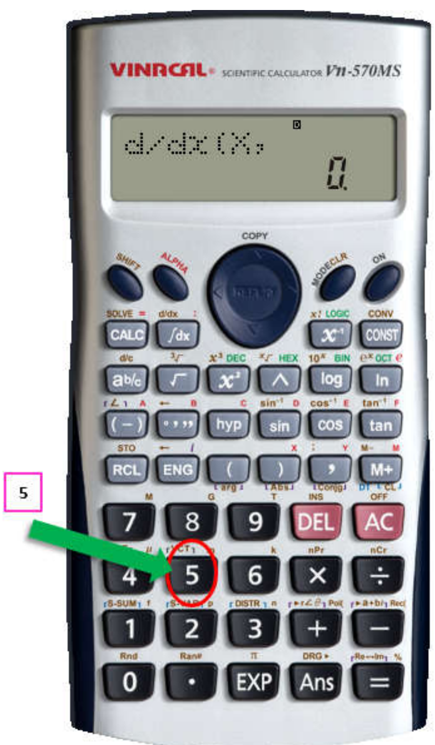


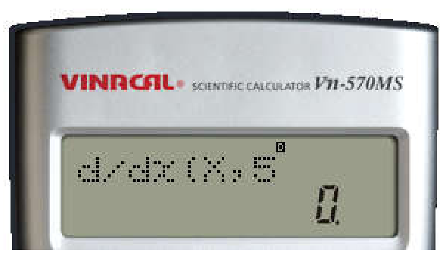
STEP 3 : Press “**,**” to separate between the function and the value that will be evaluated.



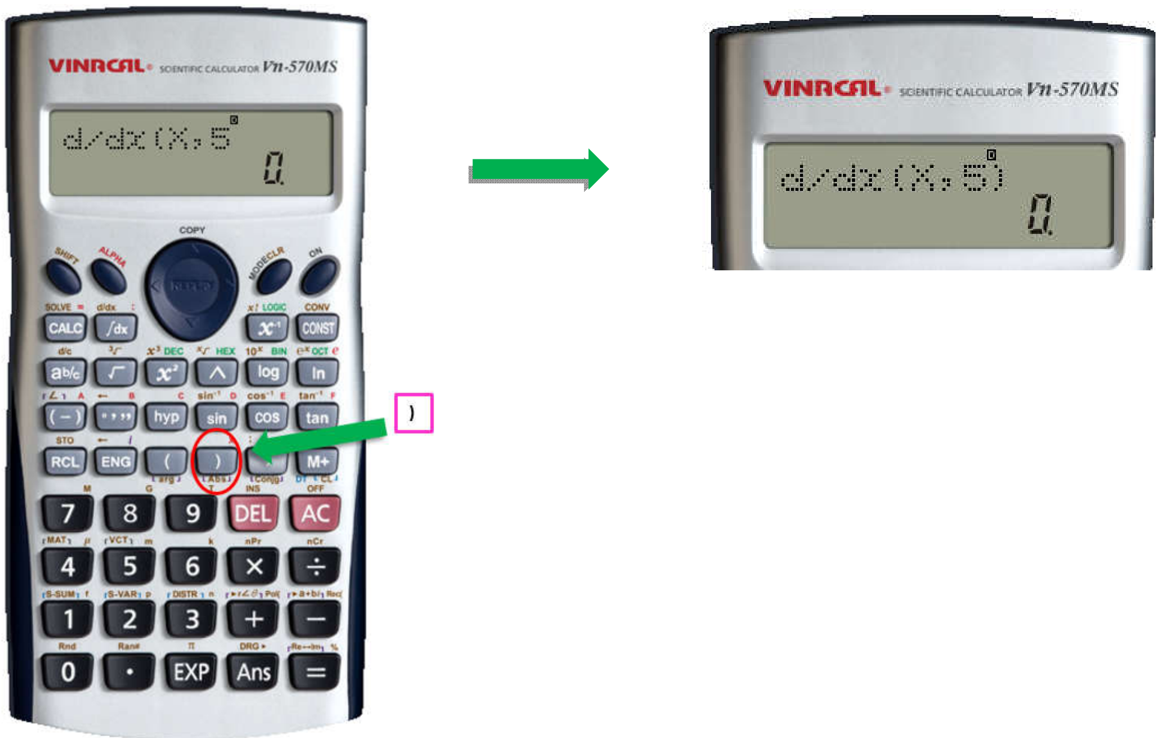


STEP 4 : Enter a particular value of *x*.

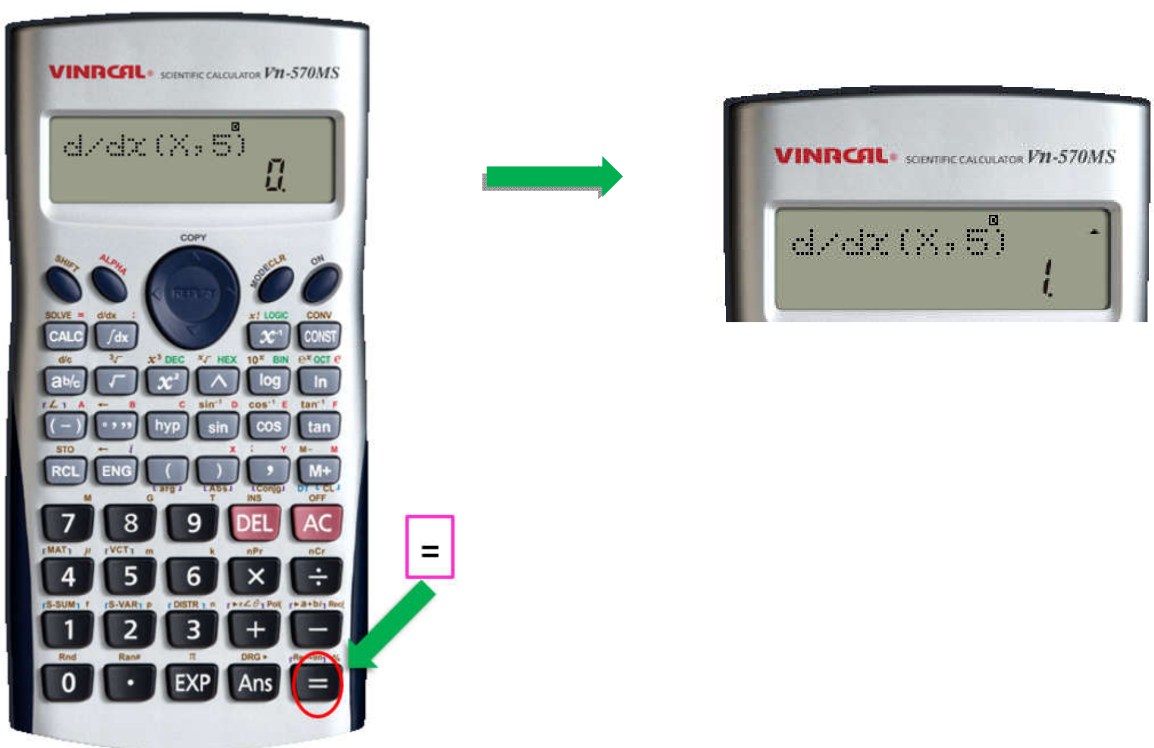




STEP 5 : Press “**)**” to end the expression and indicate that all the inputs have been inserted.



STEP 6 : Press “**=**” to obtain the output of the derivative of the function at the specific value.



GETTING STARTED WITH EVALUATING DEFINITE INTEGRALS OF A SINGLE VARIABLE

There are three inputs required to compute definite integrals using the feature provided by the 570MS calculator:

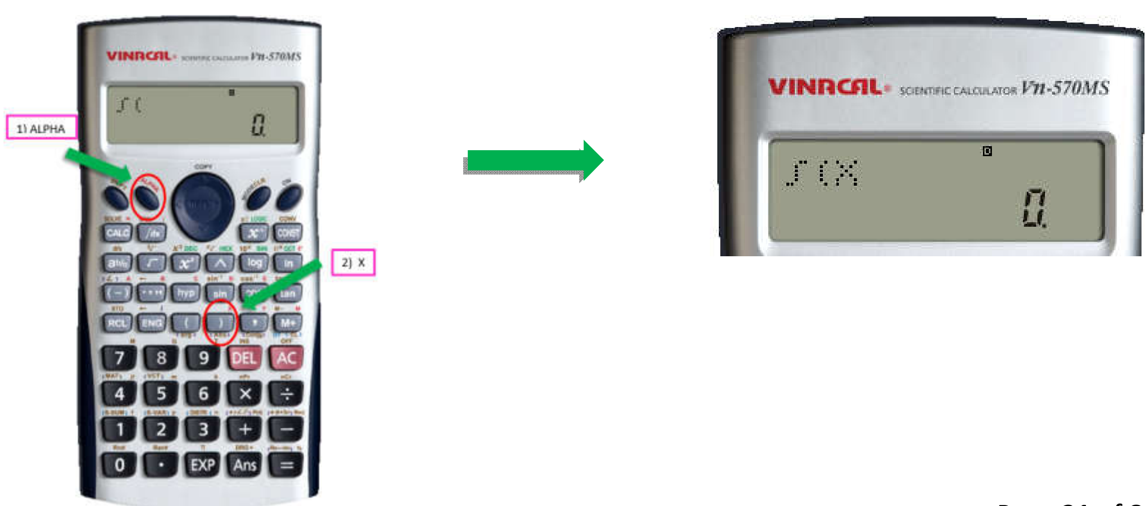
a) a function of variable *x*
b) the lower limit of the definite integral
c) the upper limit of the definite integral

$$\int_{\text{lower limit}}^{\text{upper limit}} (\text{a function})\, dx$$

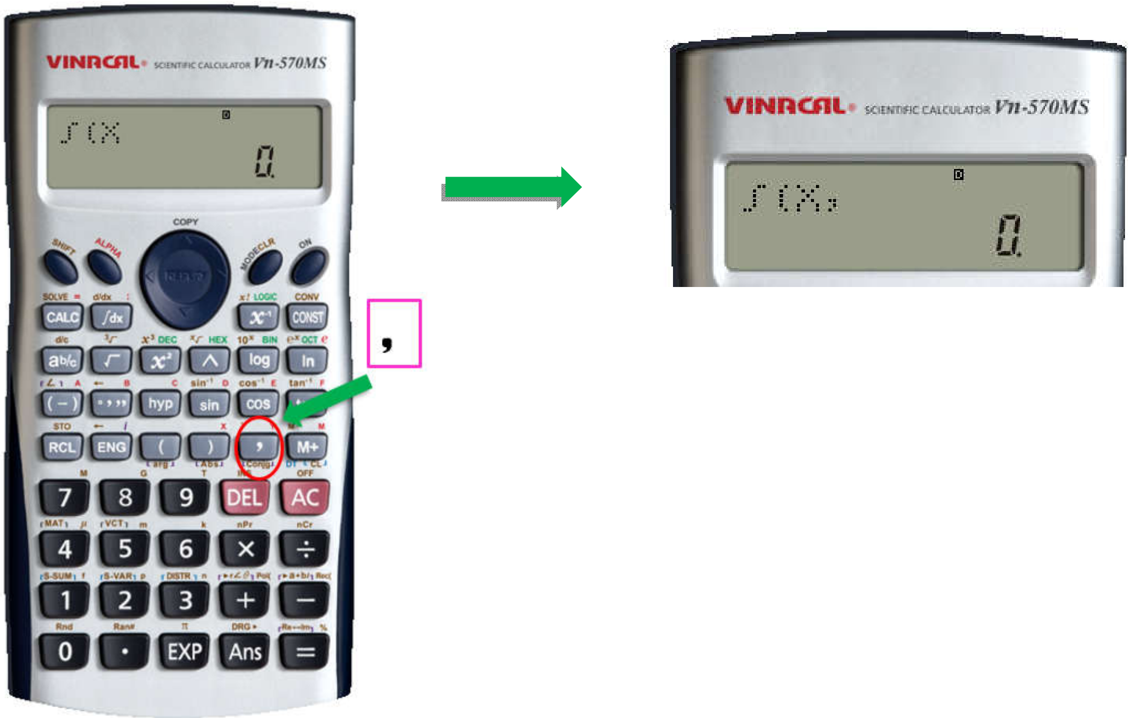
Format: $\boxed{\int dx}$ *a function* $\boxed{,}$ *the lower limit* $\boxed{,}$ *the upper limit* $\boxed{)}$ $\boxed{=}$

STEP 1 : Press "$\int$ ***dx***" to activate the integral operator.

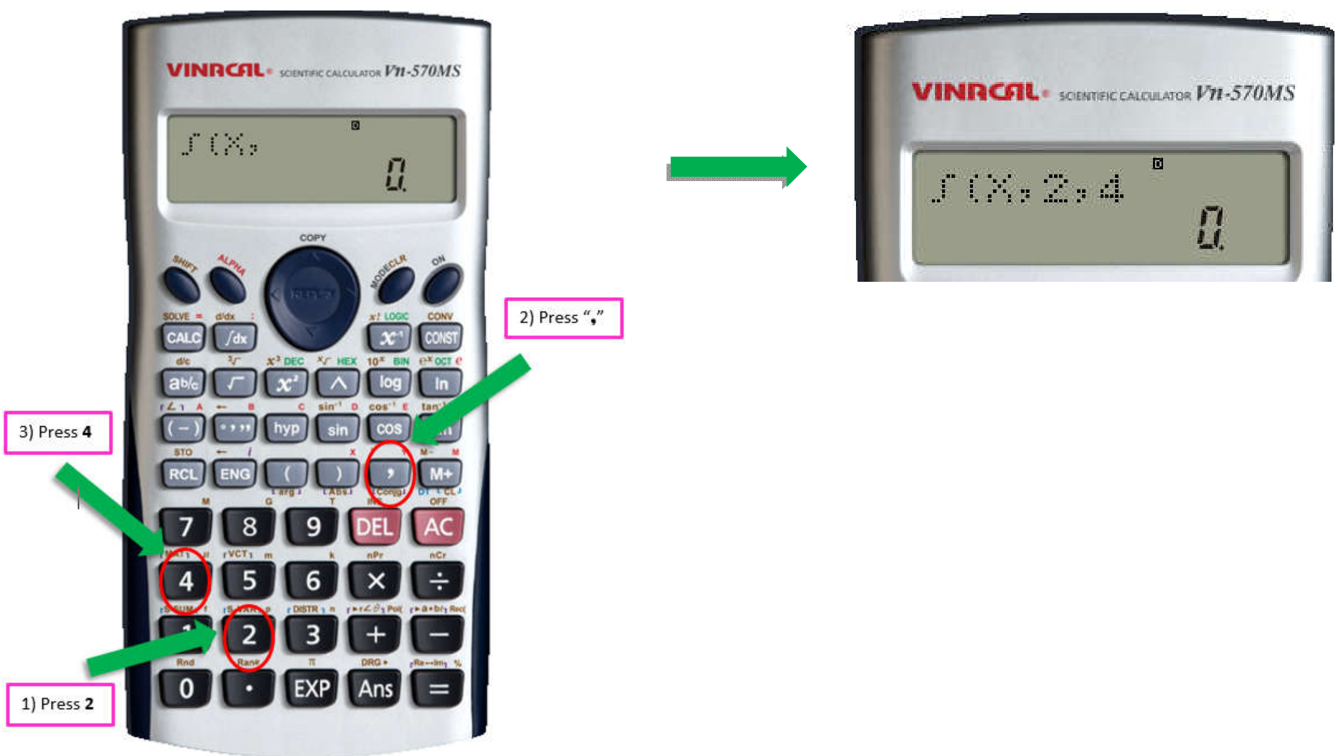
STEP 2 : Enter a function of *x*. If the function consists of the variable *x*, we may need to press "**ALPHA**", then "**)**" to key in the variable.



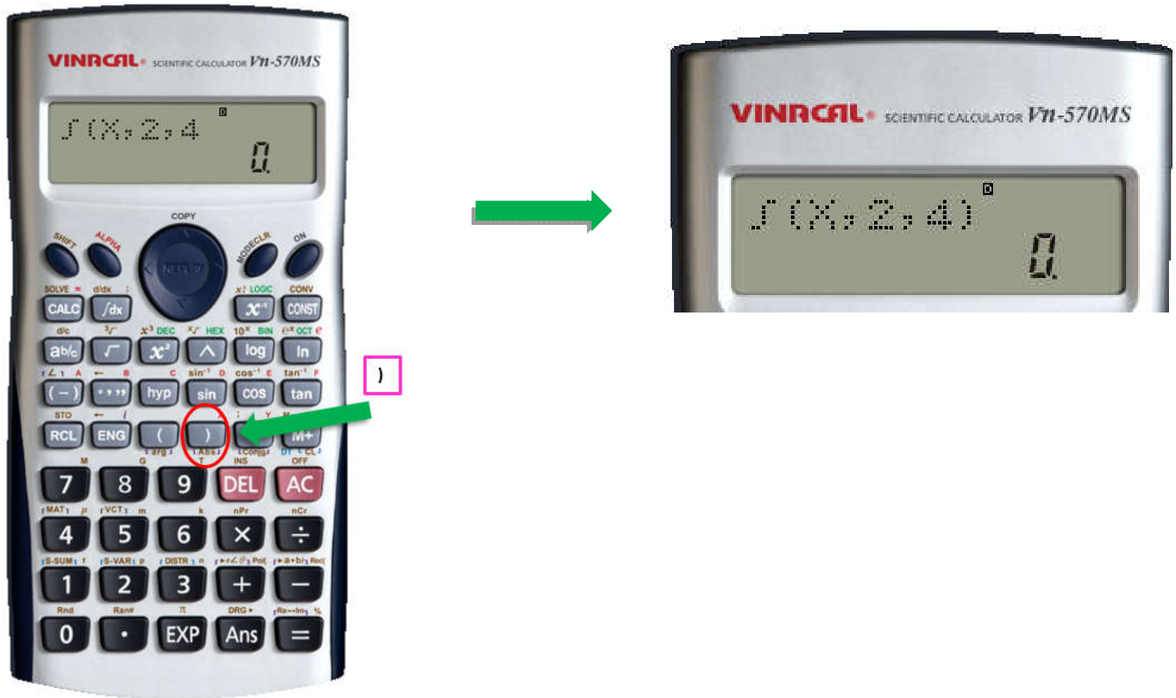
STEP 3 : Press "**,**" to separate between the function and the limits of the integral that will be evaluated.



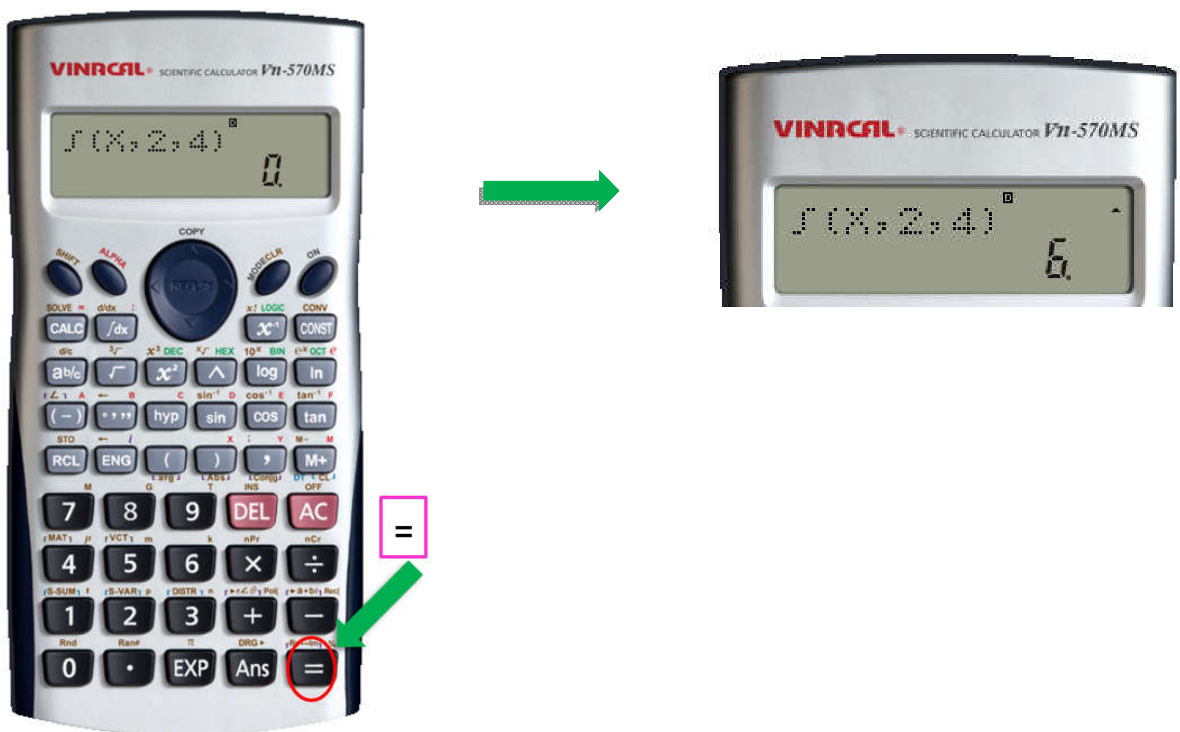
STEP 4 : Enter the values of the lower and upper limits of the integral, which separated by "**,**".



STEP 5 : Press “**)**” to end the expression and indicate that all the inputs have been inserted.



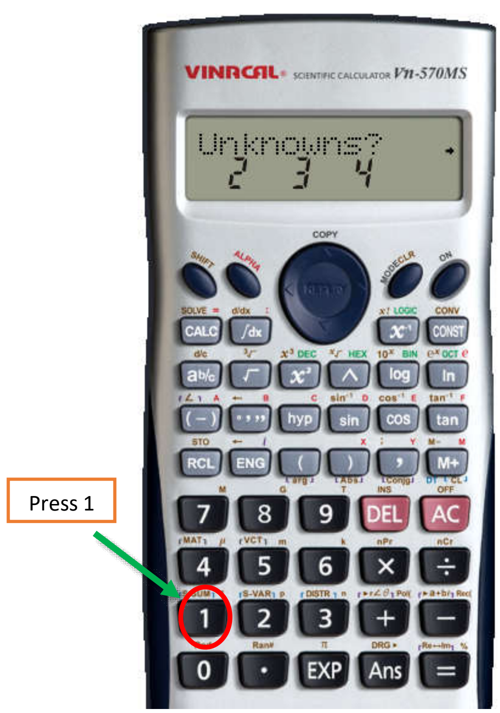
STEP 6 : Press “**=**” to obtain the output of the definite integral.



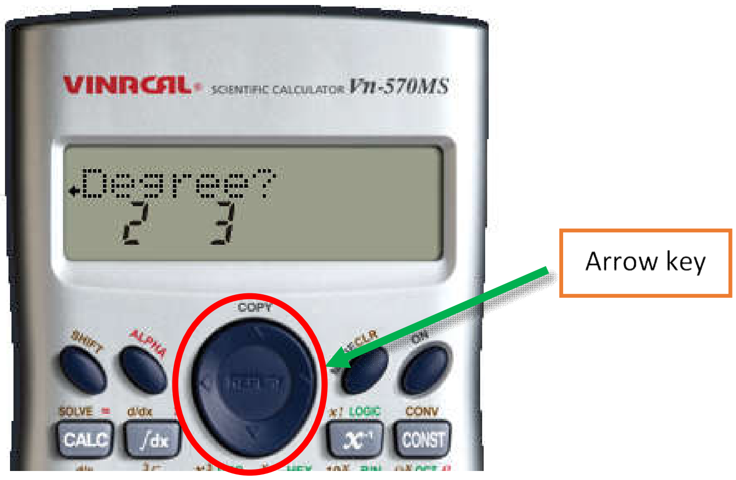
<u>GETTING STARTED WITH FACTORING A SIMPLE QUADRATIC IN THE FORM OF $ax^2 + bx + c$ USING CALCULATOR fx-570MS.</u>

STEP 1 : Press **mode** 3 times until display in the screen below appears.

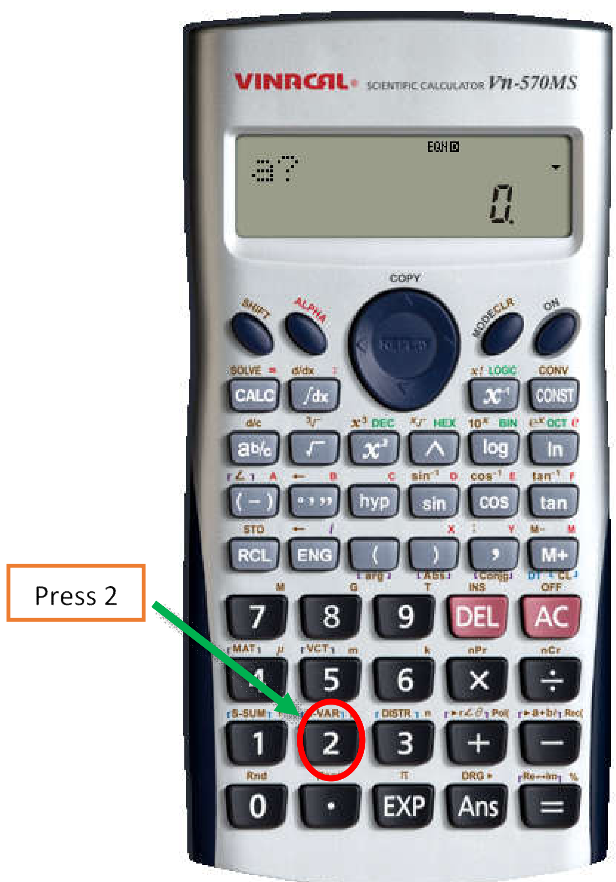
STEP 2: Press **1** to select **EQN**. You will see the display below.



STEP 3: Scroll to the right using the arrow key to select **Degree**.



STEP 4: Press **2** for quadratic degree.



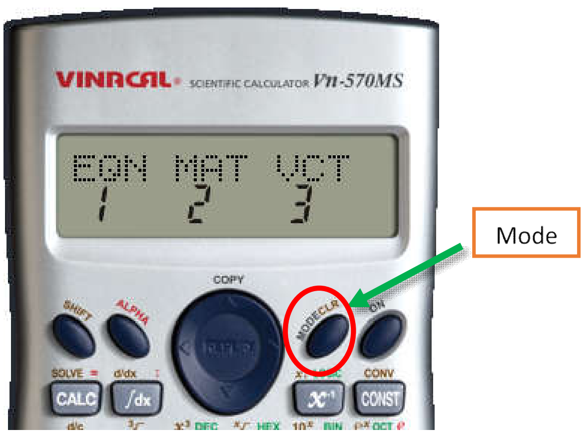
STEP 5 : Key in all the coefficients of the quadratic $ax^2 + bx + c$, i.e., a, b and c and press **=**.

STEP 6 : From the roots of the quadratic equation, i.e., x_1 and x_2, write the expression $ax^2 + bx + c$ in factored form $(x - x_1)(x - x_2)$.

GETTING STARTED WITH SOLVING A SYSTEM OF LINEAR EQUATIONS USING CALCULATOR fx-570MS.

Linear systems with two variables	Linear systems with three variables
$a_1x + b_1y = c_1$ $a_2x + b_2y = c_2$	$a_1x + b_1y + c_1z = d_1$ $a_2x + b_2y + c_2z = d_2$ $a_3x + b_3y + c_3z = d_3$

STEP 1 : Press **MODE** 3 times and press **1** to select **EQN**. You will see the display below.



STEP 2: Press **2, 3** or **4** depending on the number of **Unknowns** in the system of linear equations.

STEP 3: Key in all the coefficients of the systems of equations.

For instance, $3x + 2y = 4$
$5x - 2y = 12$

Key in $a_1 = 3, b_1 = 2, c_1 = 4$ and $a_2 = 5, b_2 = -2, c_2 = 12$.

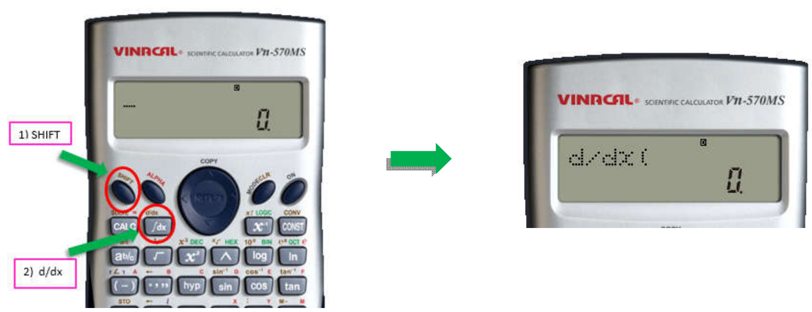
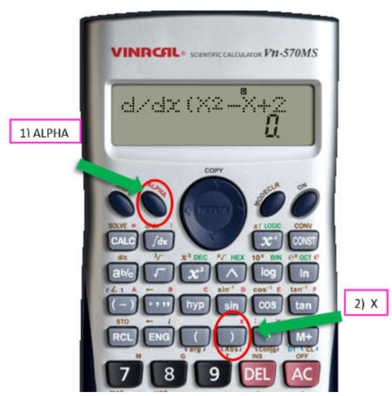
STEP 4 : Press **=** to get the values of x and y.

EVALUATING DERIVATIVES AT A SPECIFIC VALUE

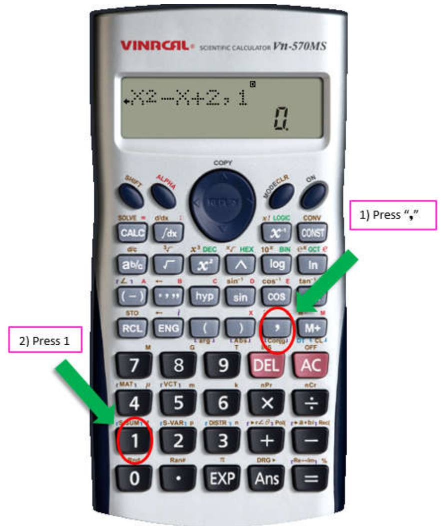
example

Evaluate $\left.\dfrac{d}{dx}\left[x^2 - x + 2\right]\right|_{x=1}$ using the scientific calculator 570MS:

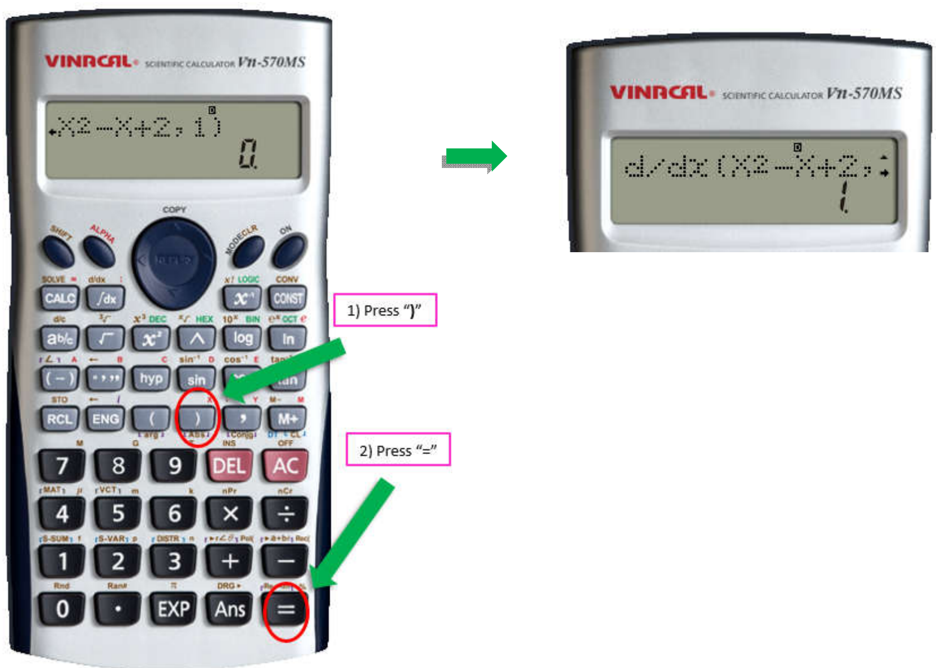
Solution :

Instruction	Figures
1. Press **SHIFT**, then **d / dx** .	
2. Enter the function $x^2 - x + 2$; we need to press "**ALPHA**"**,** then "**)**" to insert the variable *x*.	

3. Press “,” and followed by the value 1.



4. Press “)” , then “=” to obtain the result.

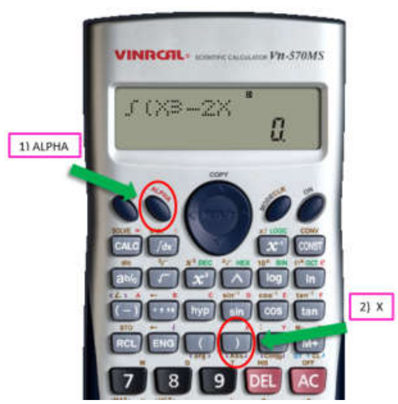
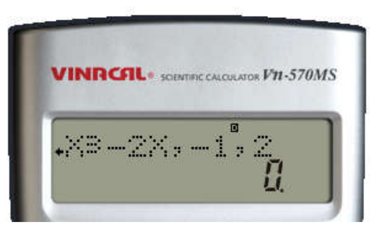


EVALUATING DEFINITE INTEGRALS OF A SINGLE VARIABLE

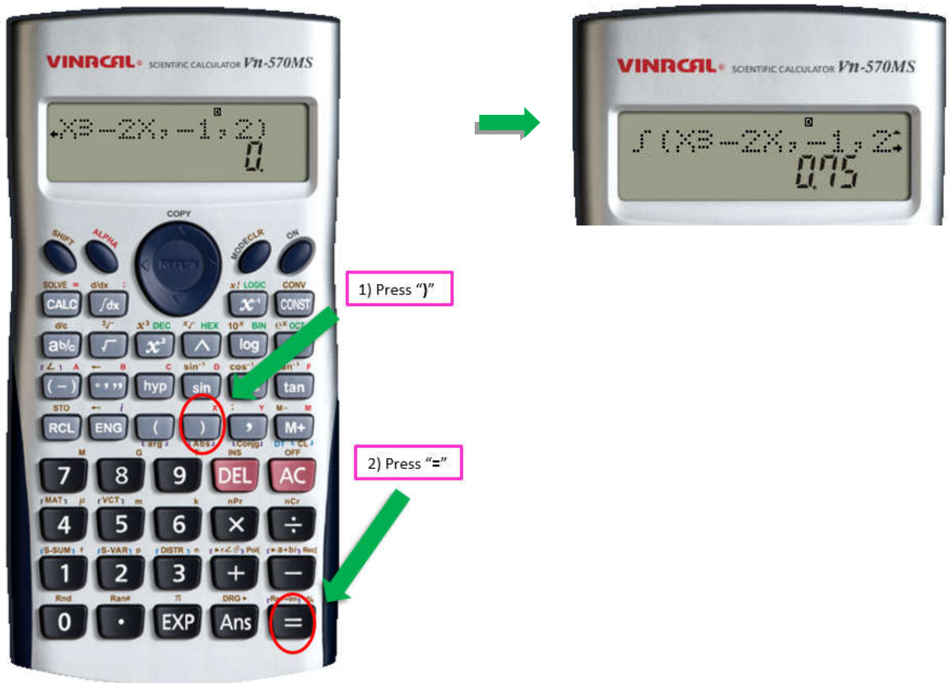
example

Evaluate $\int_{-1}^{2}\left(x^3 - 2x\right)\,dx$ using the scientific calculator 570MS:

Solution :

Instruction	Figures
1. Press "$\int dx$".	
2. Enter the function $x^3 - 2x$; we need to press "**ALPHA**", then "**)**" to insert the variable x.	
3. Press $\boxed{,}\ \boxed{-}\ \boxed{1}\ \boxed{,}\ \boxed{2}$	

4. Press “**)**” , then “**=**” to obtain the result.



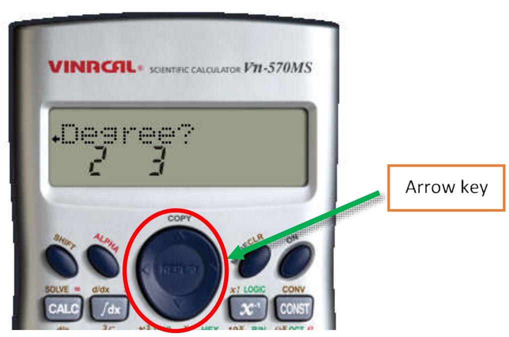
example

Factor the following quadratic polynomial using scientific calculator fx-570MS:

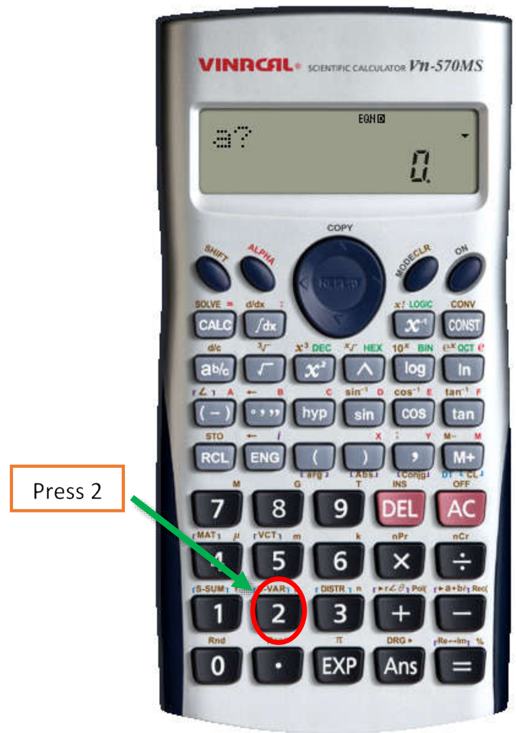
Example 1

Factor $x^2 - 8x + 12$.

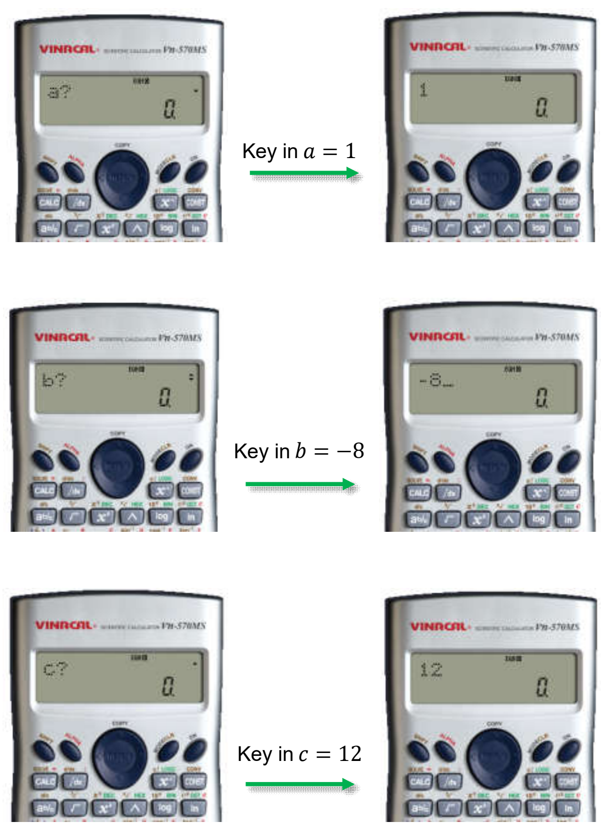
Solution :

Instruction	Input
1. Press **MODE** 3 times and press **1** for **EQN**	
2.Scroll to the right using the arrow key to select degree	

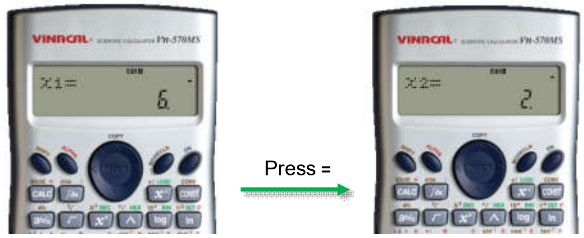
3.Press **2** for quadratic degree



4.Key in $a = 1$, $b = -8$, $c = 12$ and press **=**



5. Value of x_1 is shown in the display. To get value of x_2, press **=**



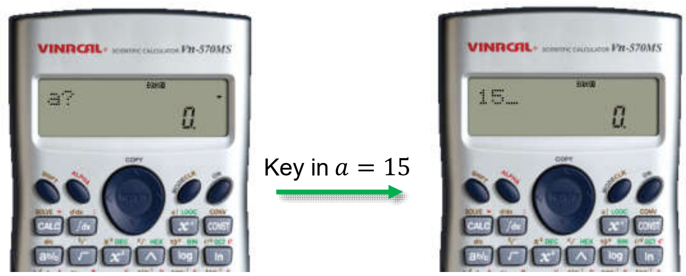
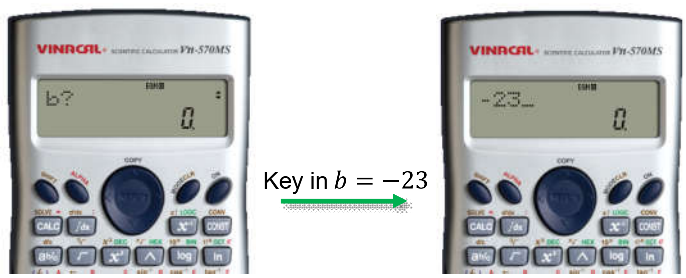
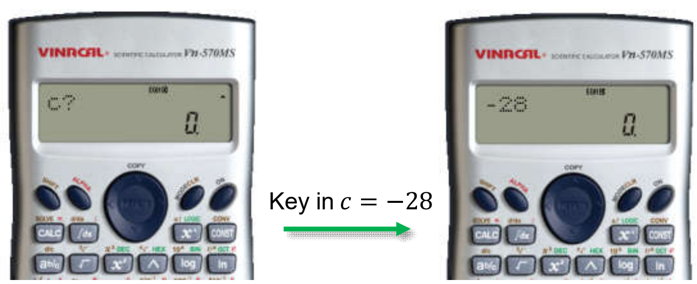
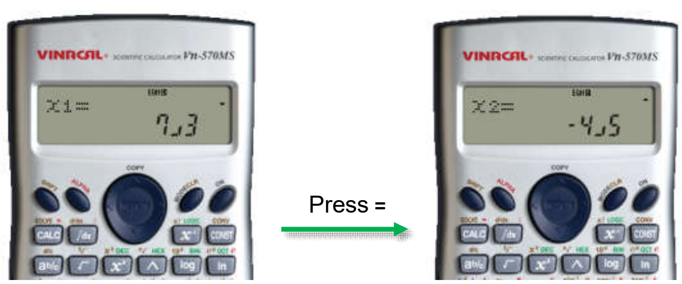
6. Write the quadratic expression in factored form.

$$x^2 - 8x + 12 = (x - 6)(x - 2)$$

Example 2

Factor $15x^2 - 23x - 28$.

Solution:

Instruction	**Input**
1. Repeat step **1-3** in **Example 1**	
2. Key in $a = 15$, $b = -23$, $c = -28$ and press **=**	  
3 Value of x_1 is shown in the display. To get value of x_2, press **=**	

4. Write the quadratic expression in factored form.

$$15x^2 - 23x - 28 = \left(x - \frac{7}{3}\right)\left(x + \frac{4}{5}\right)$$

$$= (3x - 7)(5x + 4)$$

SOLVING A SYSTEM OF LINEAR EQUATIONS

example

Solve the following systems of linear equations using scientific calculator fx-570MS.

Example 1

Solve $2x - 5y = 15$

$$3x + \ y = 31$$

Instruction	Input
1. Press **MODE** 3 times and press **1** for **EQN**	
2. Press **2** for 2 **Unknowns**, i.e., x and y	

3. Key in values and press **=**

$a_1 = 2,$
$b_1 = -5,\ c_1 = 15$

$a_2 = 3,$
$b_2 = 1,\ c_2 = 31$

5. Value of x is shown in the display. To get value of y, press **=**

Example 2

Solve
$$\begin{aligned} 2x + \ y + 2z &= -2 \\ -2x + 2y - \ z &= -5 \\ 4x + \ y - 2z &= \ 0 \end{aligned}$$

Instruction	Input
1. Press **MODE** 3 times and press **1** for **EQN**	
2. Press **3** for 3 **Unknowns**, i.e., x, y and z	
4. Key in values and press **=**	   

$a_1 = 2,\ b_1 = 1,$

$c_1 = 2, d_1 = -2$

$a_2 = -2,\ b_2 = 2,$

$c_2 = -1, d_2 = -5$

$a_3 = 4,\ b_3 = 1,$

$c_3 = -2, d_3 = 0$

5. Value of x is shown in the display. To get value of y and z, press **=**

EXERCISE

Evaluate the derivatives of the following functions at the given points using scientific calculator.

Question 1

$$\frac{d}{dx}\left[\frac{\sin x-1}{\cos x+1}\right]\bigg|_{x=1}$$

Question 2

$$\frac{d}{dx}\left[\frac{e^{x^2}}{2}+\ln(x+2)\right]\bigg|_{x=0}$$

Question 3

$$\frac{d}{dx}\left[\left(x+\cos^2 x\right)^5\right]\bigg|_{x=0}$$

Question 4

$$\frac{d}{dx}\left[\frac{x}{\sqrt{3-2x}}\right]\bigg|_{x=1}$$

Answers: 1) 0.2946 2) 0.5 3) 5 4) 2

EXERCISE

Evaluate the following definite integrals using scientific calculator.

Question 1

$$\int_0^4 \sqrt{2x+1}\, dx$$

Question 2

$$\int_0^{\pi} e^x \sin(e^x)\, dx$$

Question 3

$$\int_0^1 \frac{x^2}{e^{2x}}\, dx$$

Question 4

$$\int_0^{3\pi/2} \sin^2(2x)\, dx$$

Answers: 1) 8.6667 2) : 0.9492 3) 0.0808 4) 2.3562

FES455

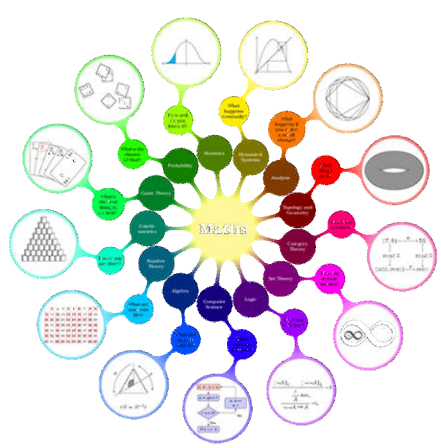
Module 3

GeoGebra 2D & 3D

Presented by

Norshuhada Samsudin

Fuziatul Norsyiha Ahmad Shukri



MODULE 3

GEOGEBRA: 2D & 3D GRAPH

PREPARED BY

NORSHUHADA BINTI SAMSUDIN & FUZIATUL NORSYIHA BINTI AHMAD SHUKRI

INTRODUCTION

GeoGebra Graphing Calculator is a dynamic mathematics software that show connection between geometry and algebra. The GeoGebra Graphing Calculator is available online. This Graphing Calculator is also available in the Google Play Store for android user and the App Store for iOS devices.

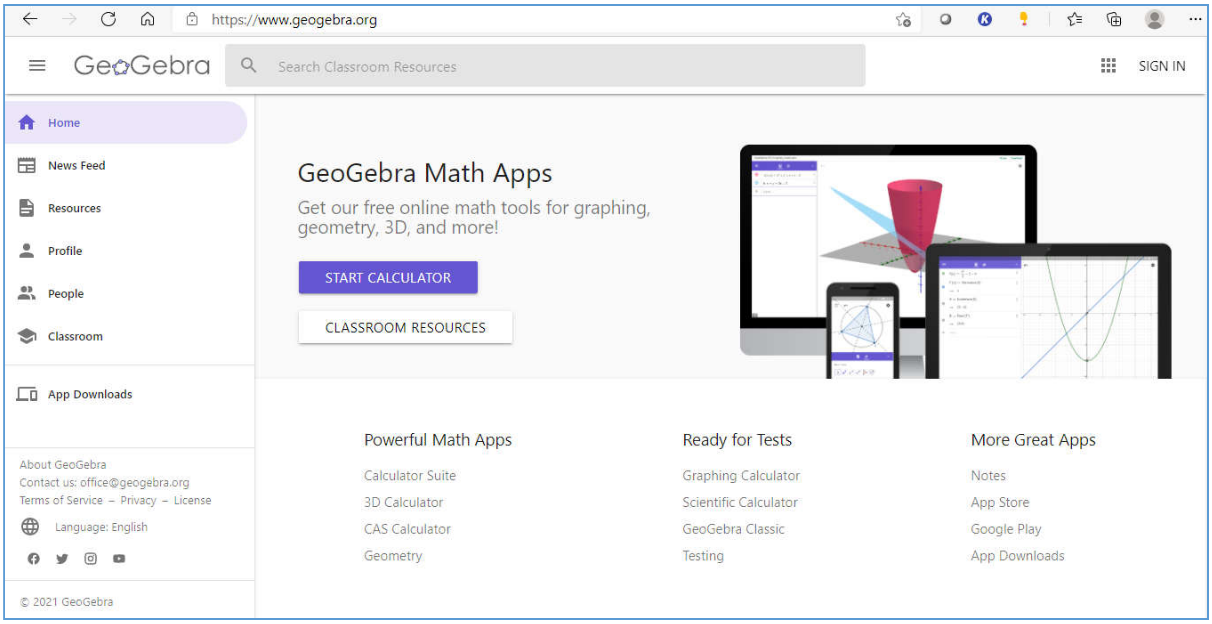
In this module, students will learn how to use the Geogebra Graphing Calculator to sketch the graph of 2D and 3D functions.

BENEFITS OF GEOGEBRA GRAPHING CALCULATOR:

- Easy to use, easy access and free.
- Makes learning activities interesting, fun, and meaningful.
- Allow students to understand more about geometry.

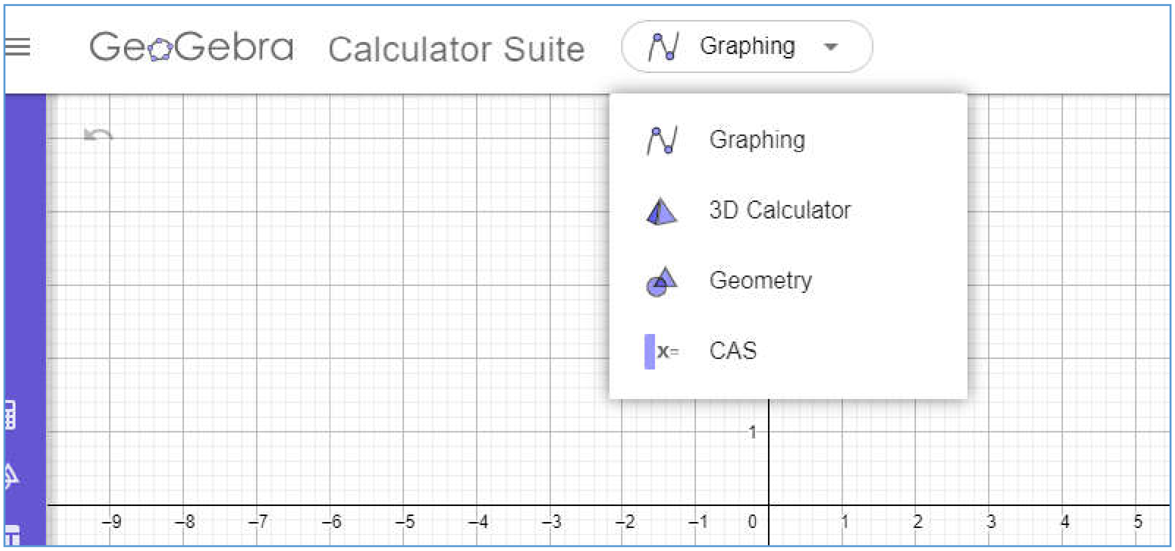
GETTING STARTED WITH GEOGEBRA-GRAPH OF 2D & 3D FUNCTIONS.

STEP 1 : Go to www.geogebra.org and the webpage below will be appeared.

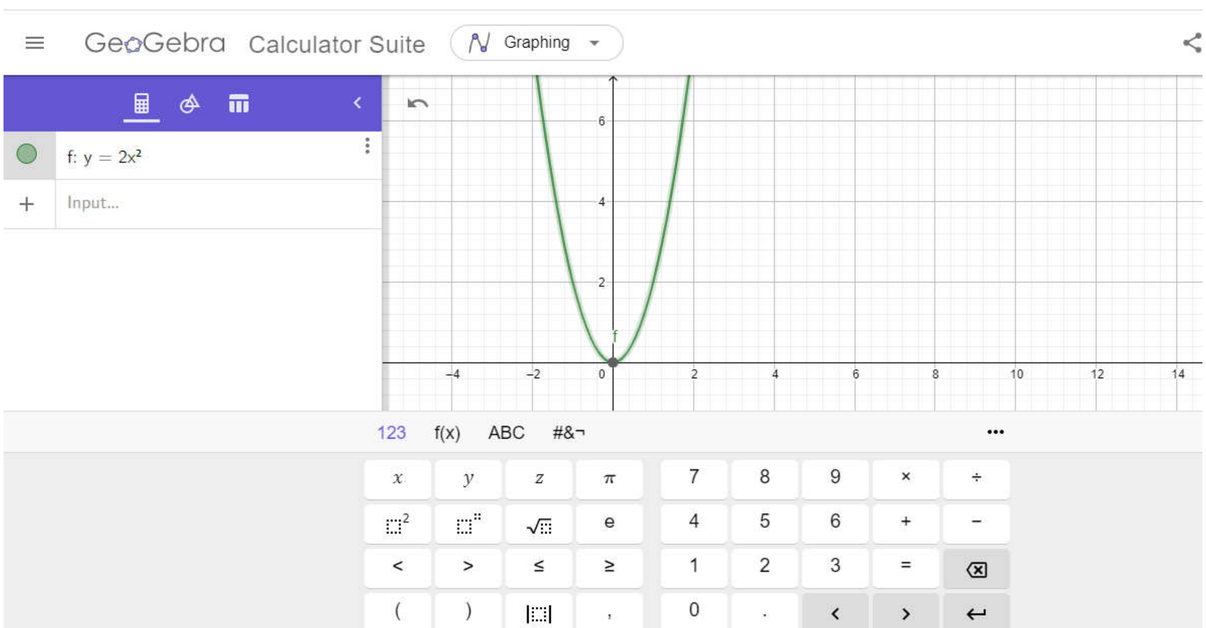


STEP 2: Click "Start Calculator" button. You will see the interface below.

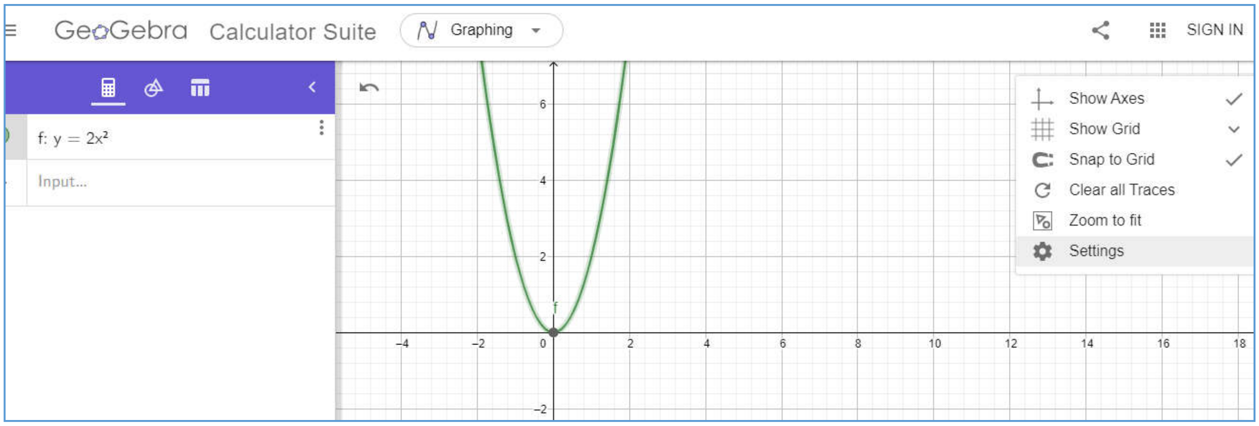
STEP 3: For 2D graph, choose **Graphing** and for 3D graph, choose **3D Calculator.**



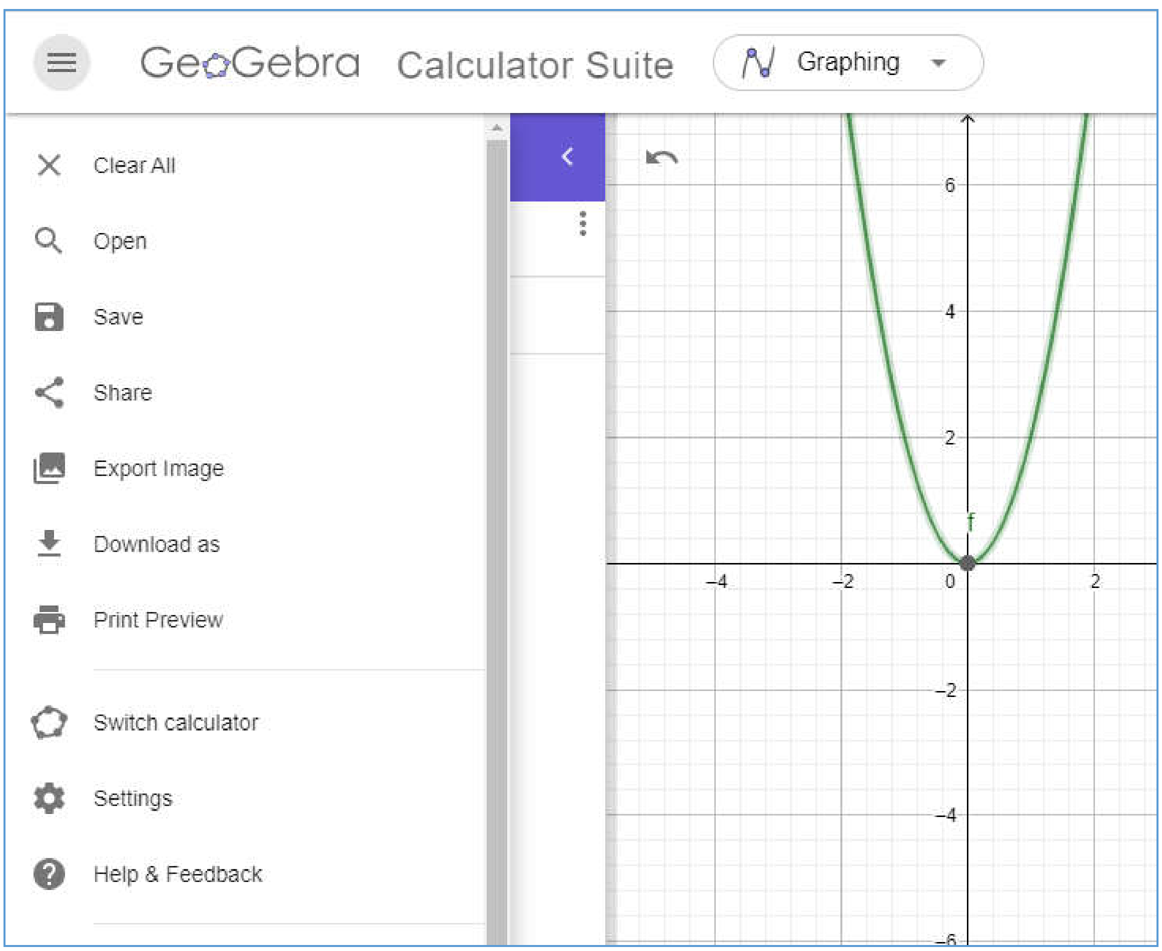
STEP 4: Type function on the input bar and the graph of that function will appear on the graphic view.



STEP 5 : Click on the "Setting" to change all the setting that related to the graph.



STEP 6 : Click on the menu bar if you want to save/export/download the graph.



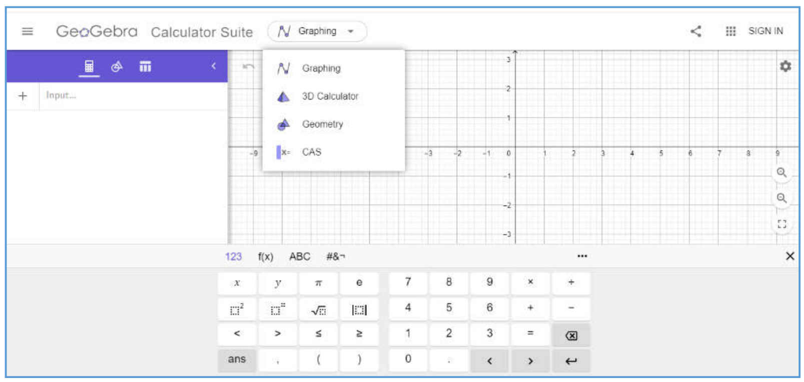
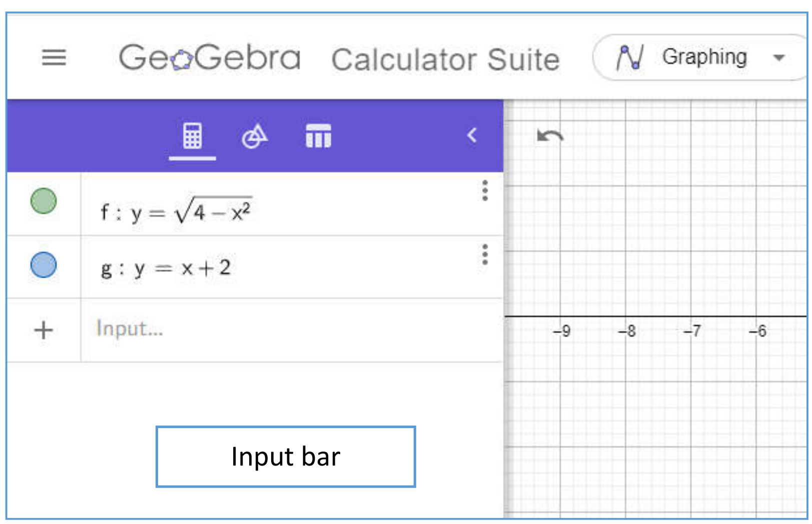
example

Sketch the graph of the following function using GeoGebra Calculator:

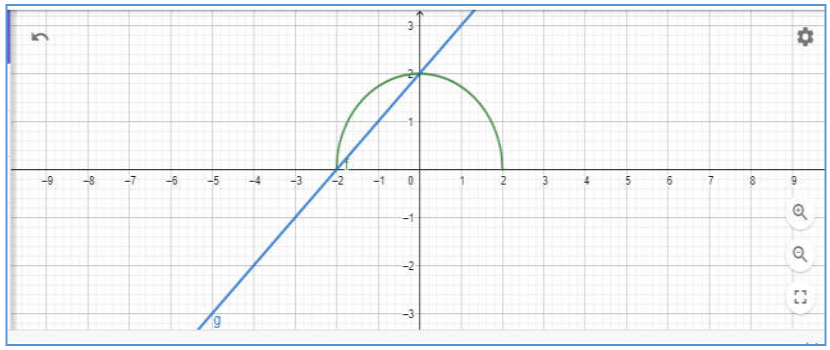
Example 1

$y = \sqrt{4 - x^2}$ and $y = x + 2$.

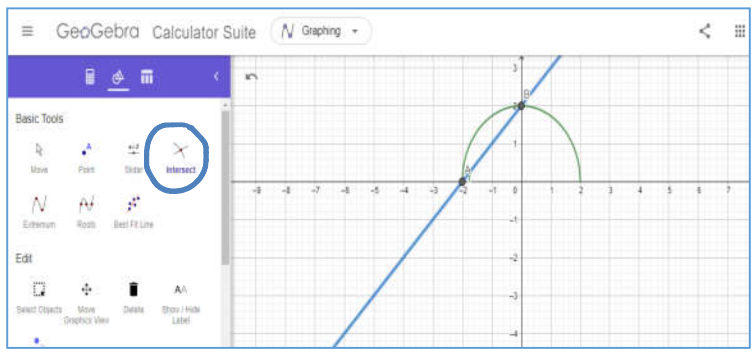
Solution :

Instruction	Input
2. Type www.geogebra.org/calculator and select Graphing.	
2.Type the function on the side bar.	

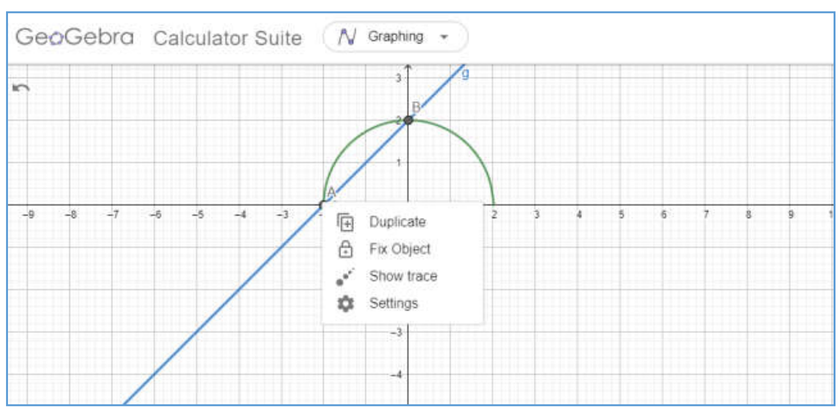
3.Graph of semicircle and line will appear on the Graphic View.



4.Click on the Tool Bar and choose Intersect. Graph will show the intersection points between these two graphs.

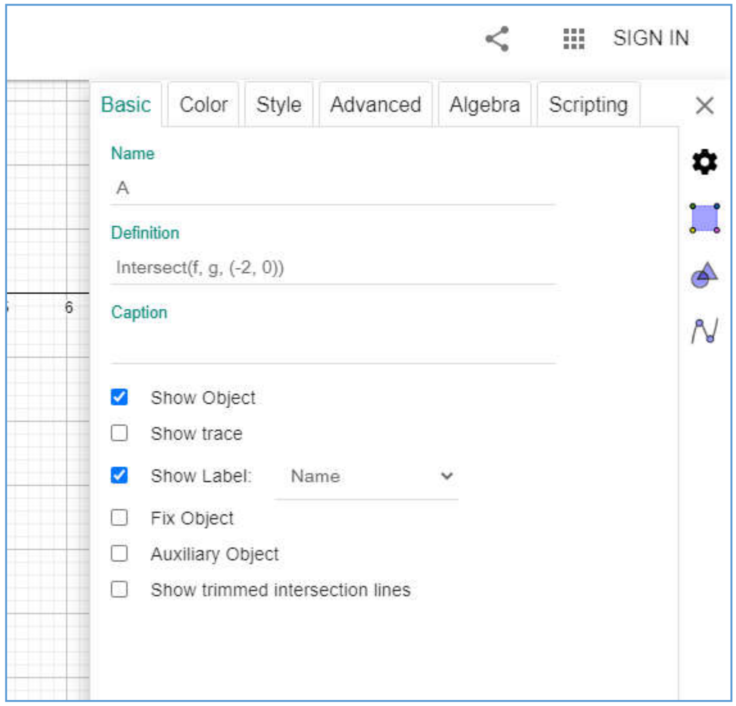


5. Click on point A, and choose setting.

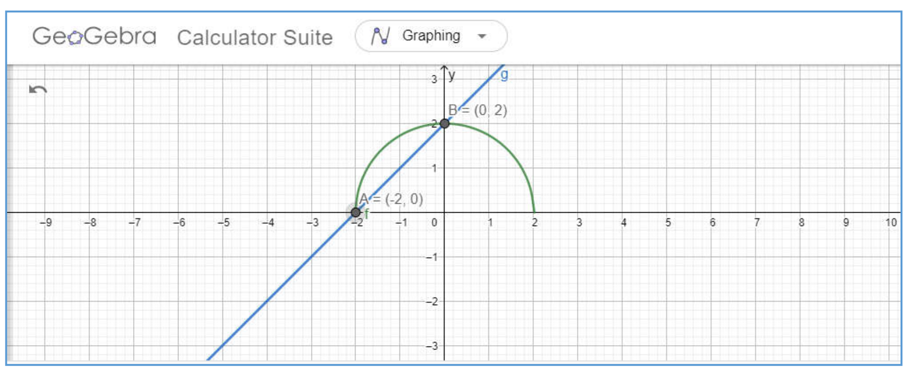


6. The setting of the point A will appear on your right-hand side.

To label point A, select Show Label and choose Name and Value.



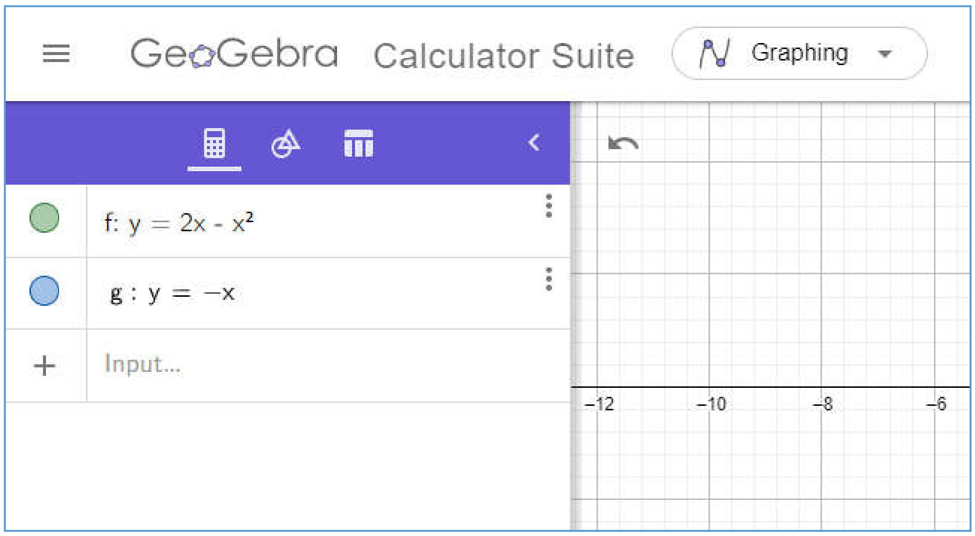
7.Repeat the same method at point B. You can also change the color of the graph on the setting.



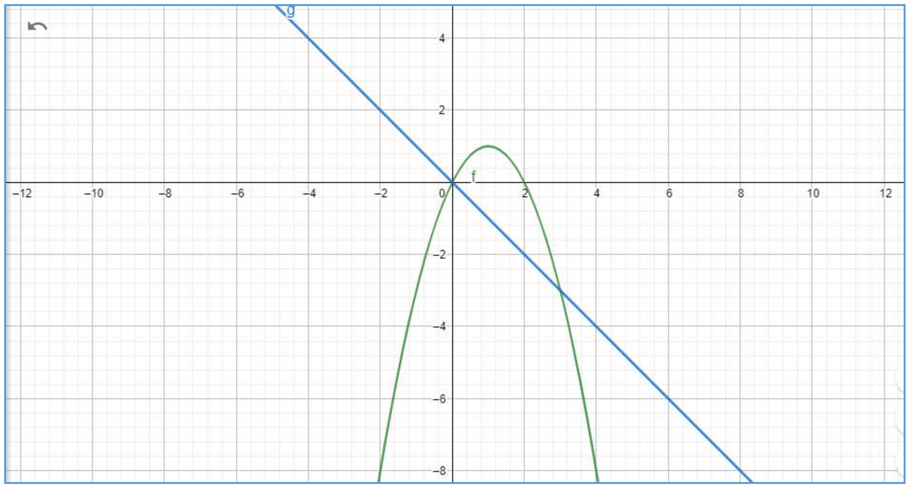
Example 2

$y = 2x - x^2$ and $y = -x$.

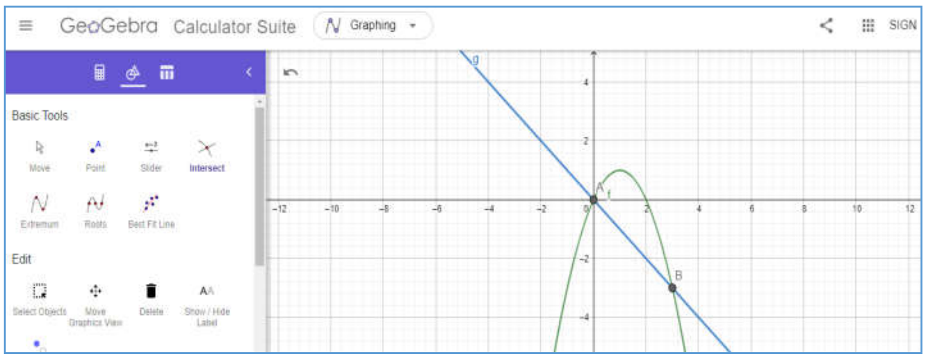
Solution

Instruction	Input
1. Type the function on the side bar.	

2.The graph of both functions will appear on the graphic view.



3.Label the intersection points by clicking on Tool Bar → Intersect.



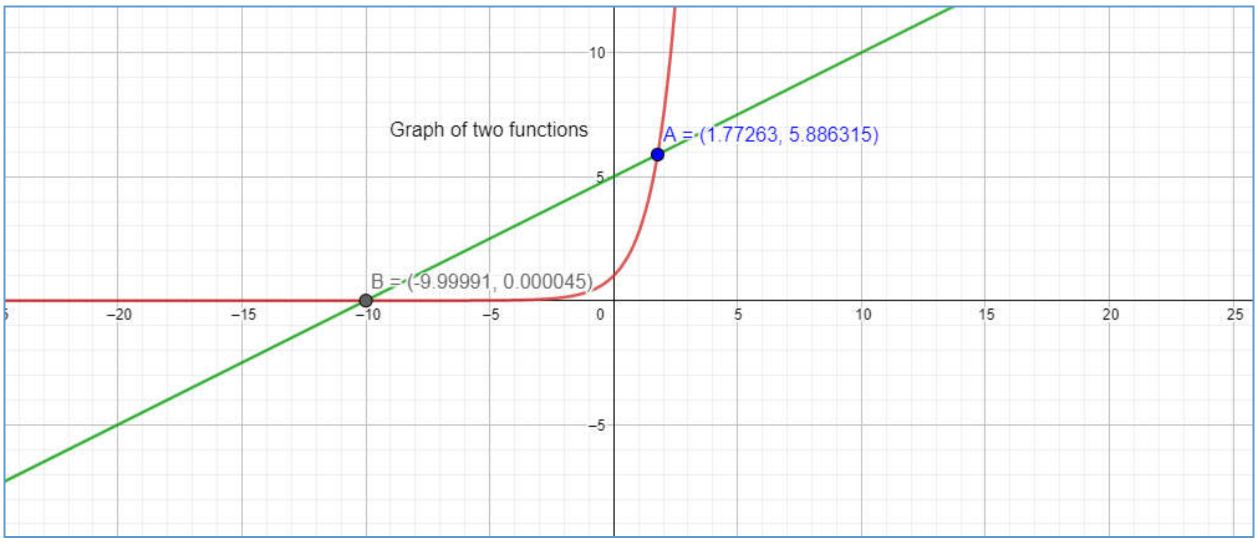
4. Change the setting of the graph like previous example.

Example 3

$y = e^x$ and $y = \frac{1}{2}x + 5$.

Solution:

Follow the same method of previous examples and you will see the graph like this appear on the Graphic View.

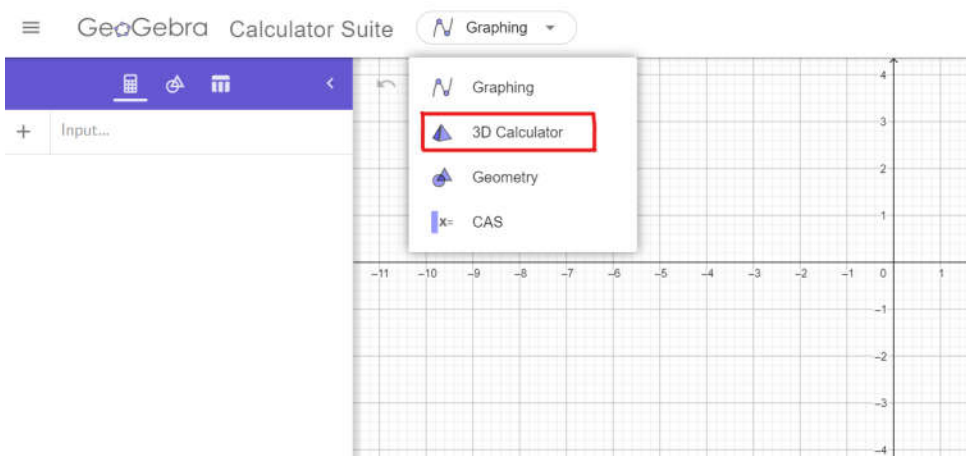


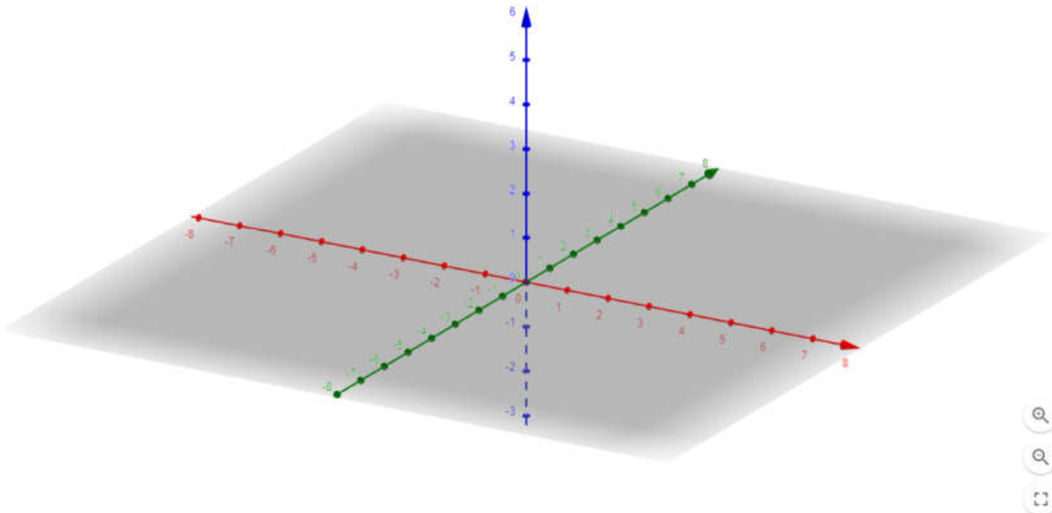
example

Example 1

Step 1:

- **Click Graphing and select 3D Calculator**



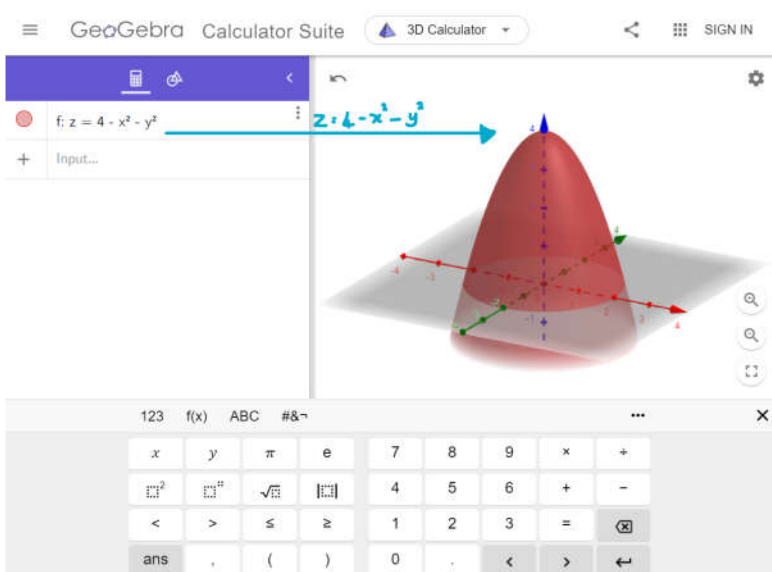


Interface of 3D Calculator

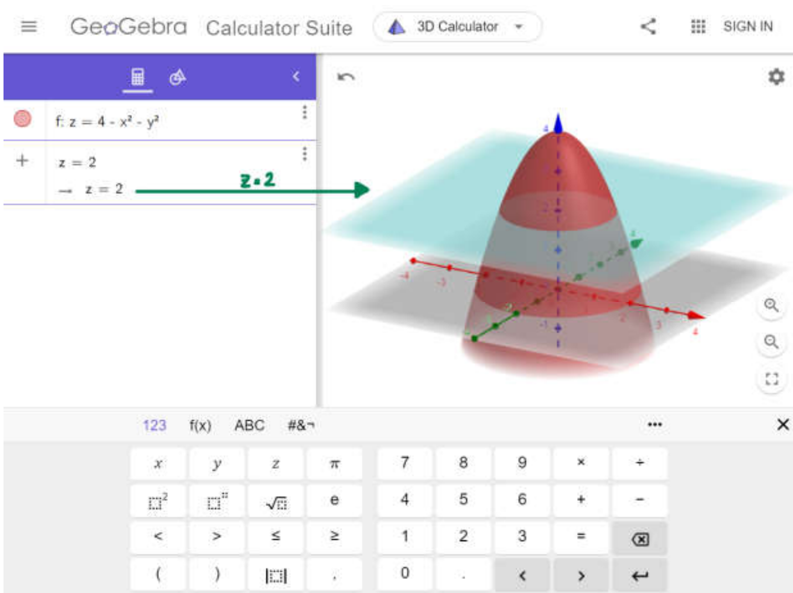
Step 2:

- **Insert the functions**

First function: $z = 4 - x^2 - y^2$



Second function: $z = 2$

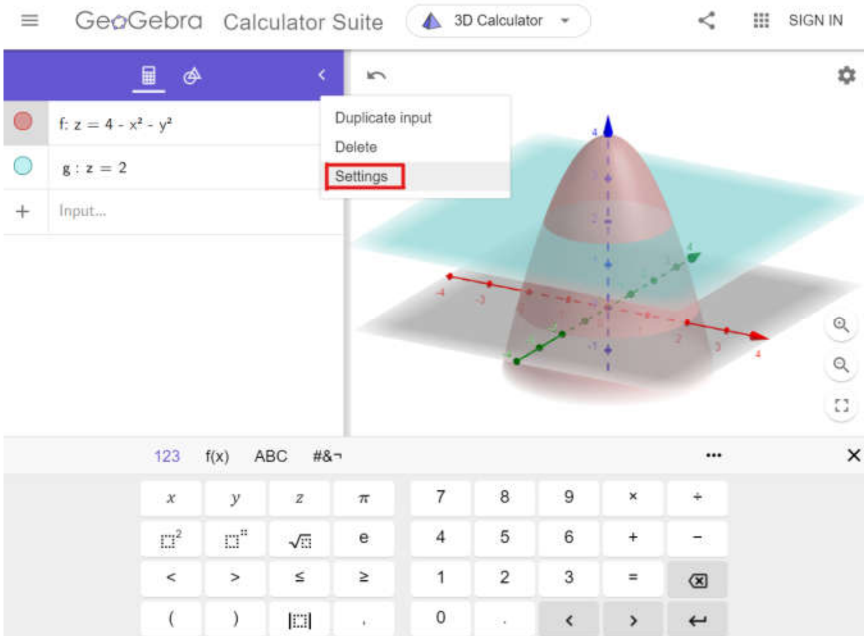


Step 3:

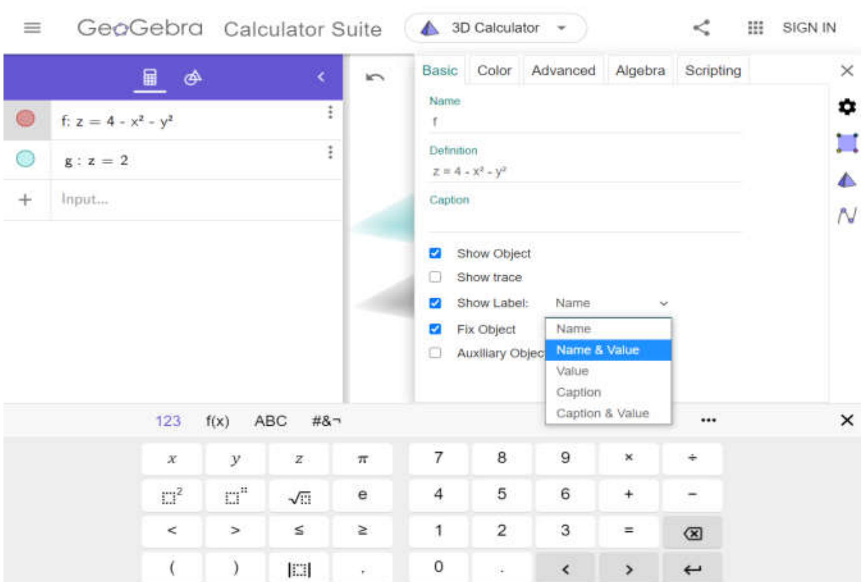
- **Change the setting**

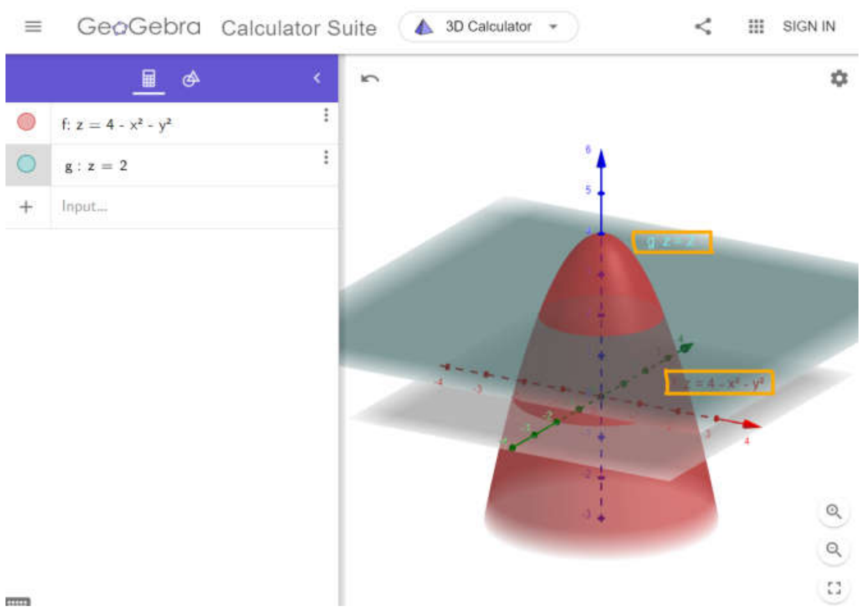
  Example: Label the functions & Axes

  1) Click setting at the right function first

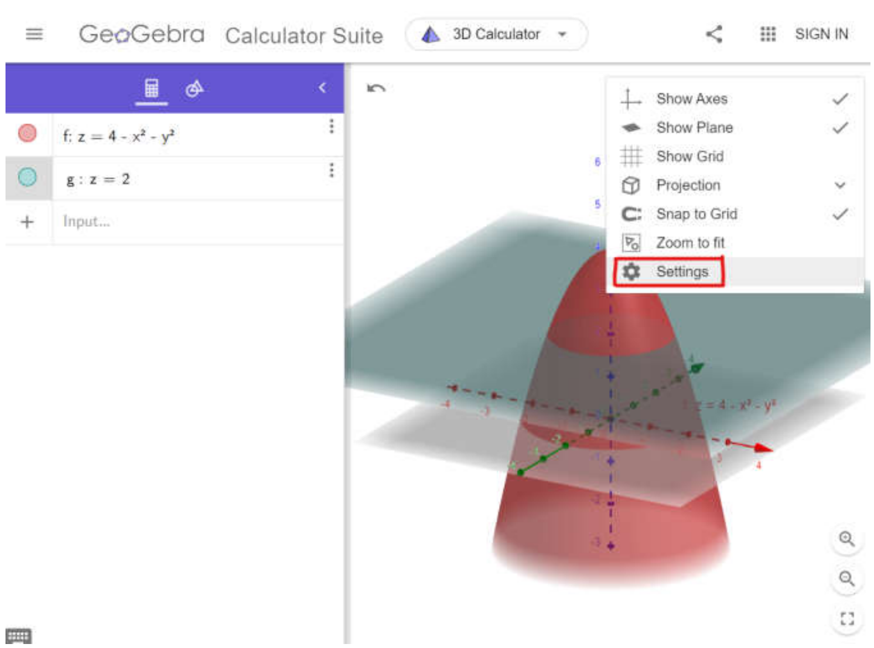


2) **Tick show label and select ‘Name & Value’**
3) **Repeat the step to the other function**

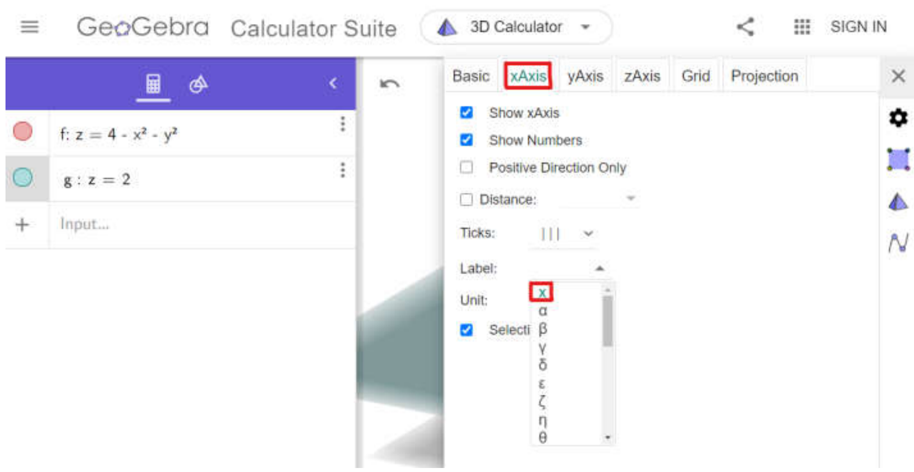


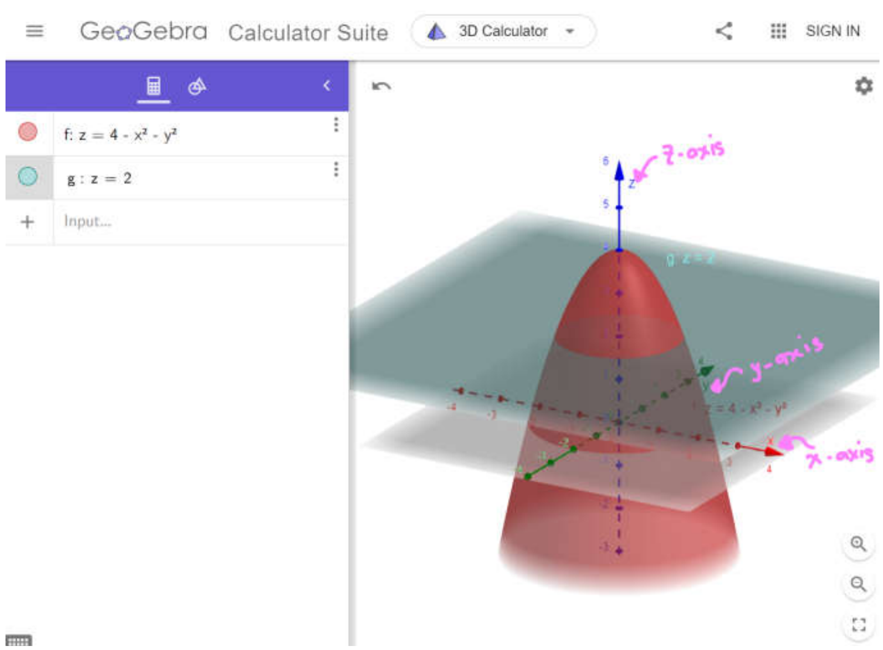


4) **Insert the axes by click symbol setting at the right top**

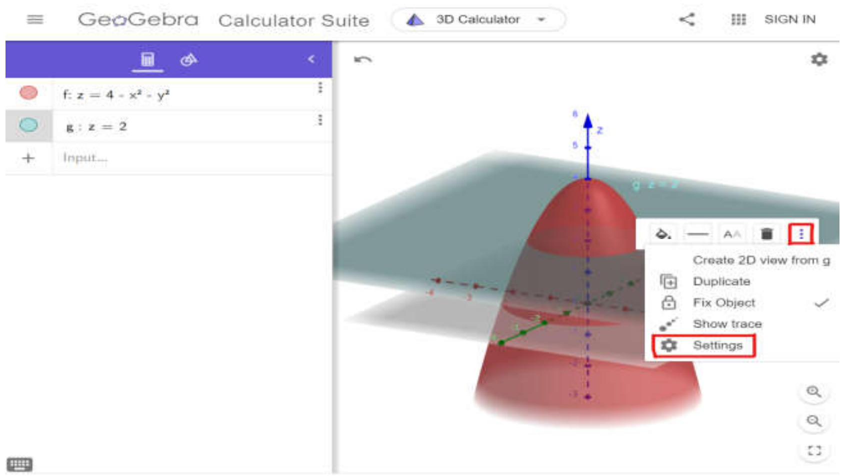


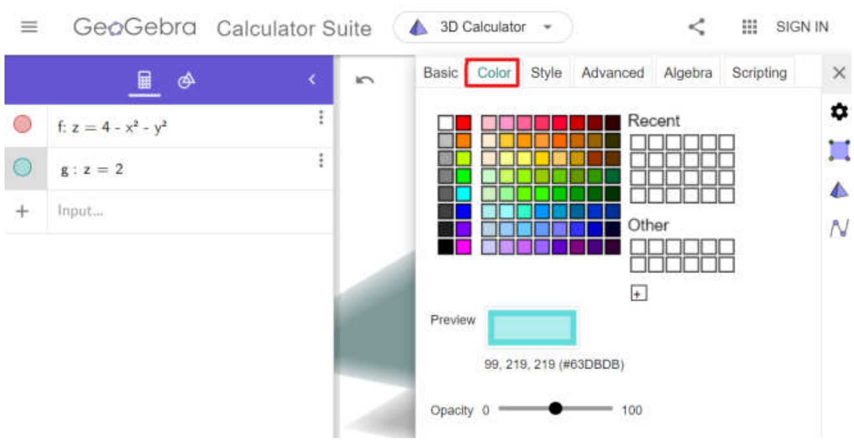
5) Click xAxis → Label → x
Repeat the step for yAxis and zAxis



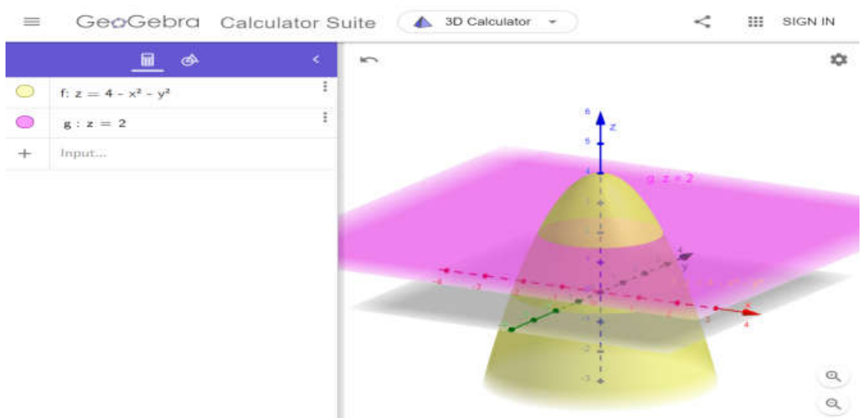


6) Change the colour of solid
Right click at the solid figure → click triple dot → setting → color





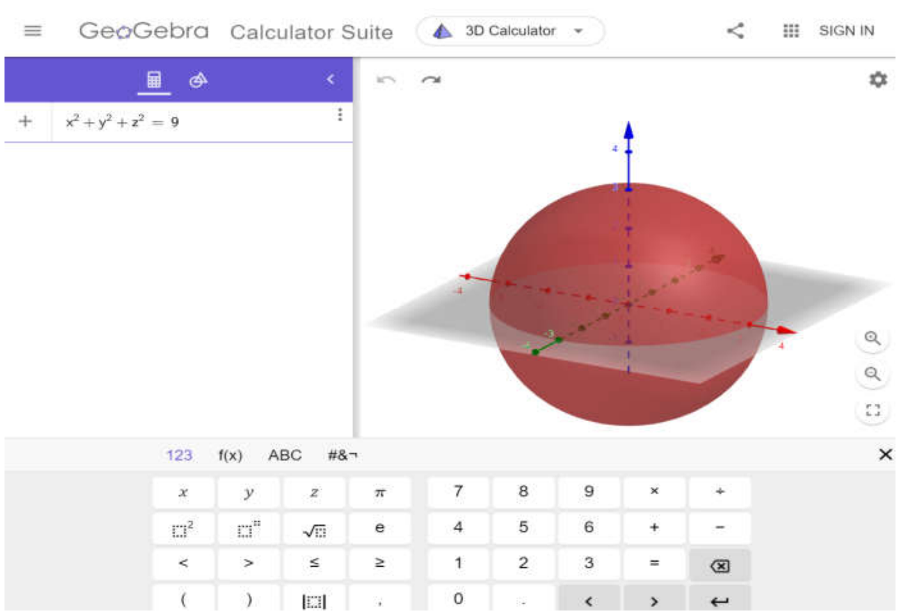
Choose the color



Example 2

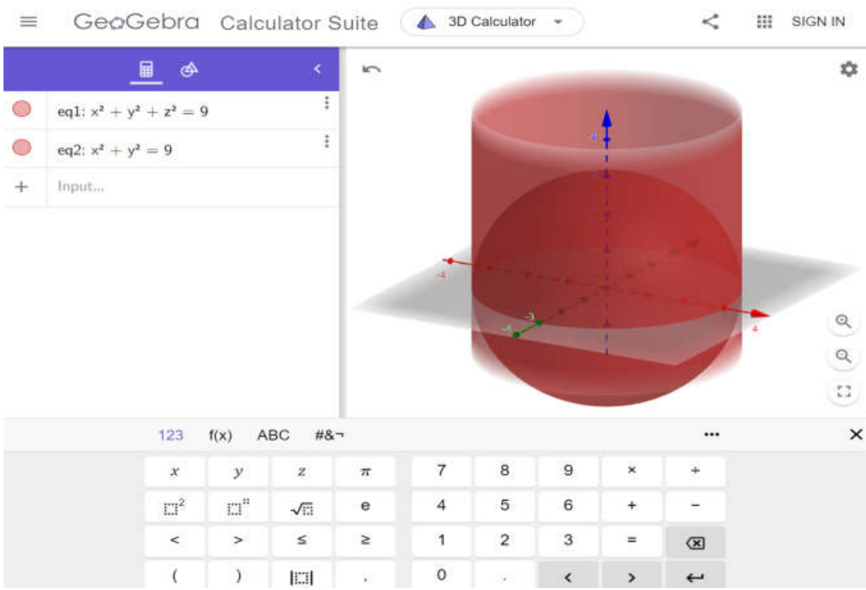
1) Insert the first function

$$x^2 + y^2 + z^2 = 9$$

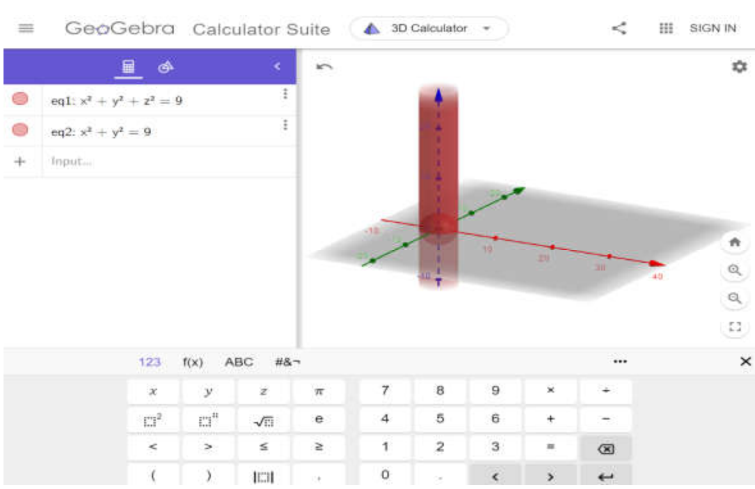


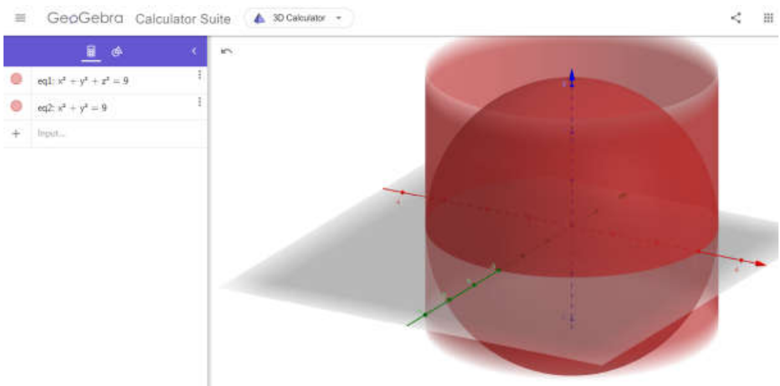
2) Insert the second function

$$x^2 + y^2 = 9$$



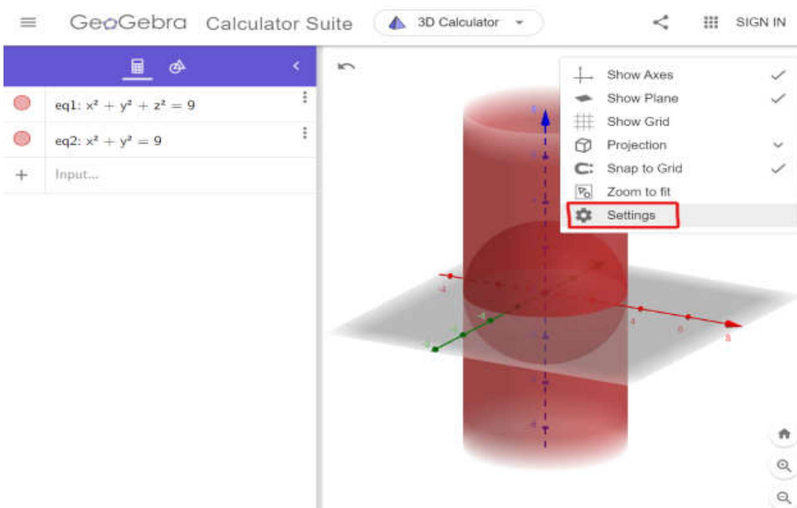
3) Zoom in and zoom out

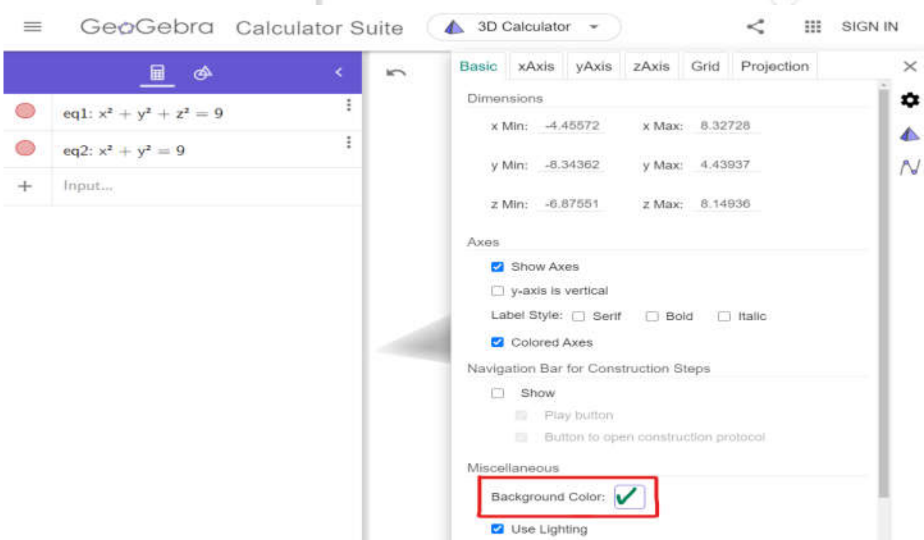


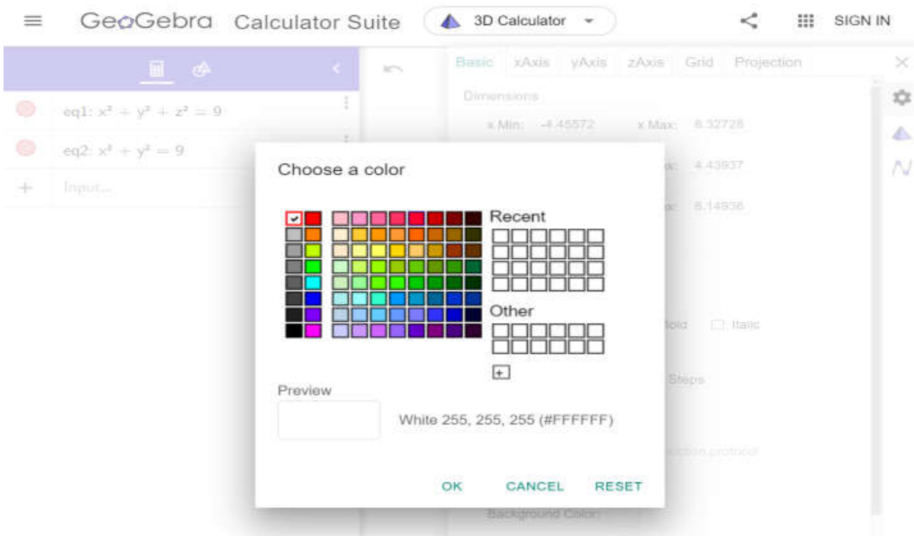


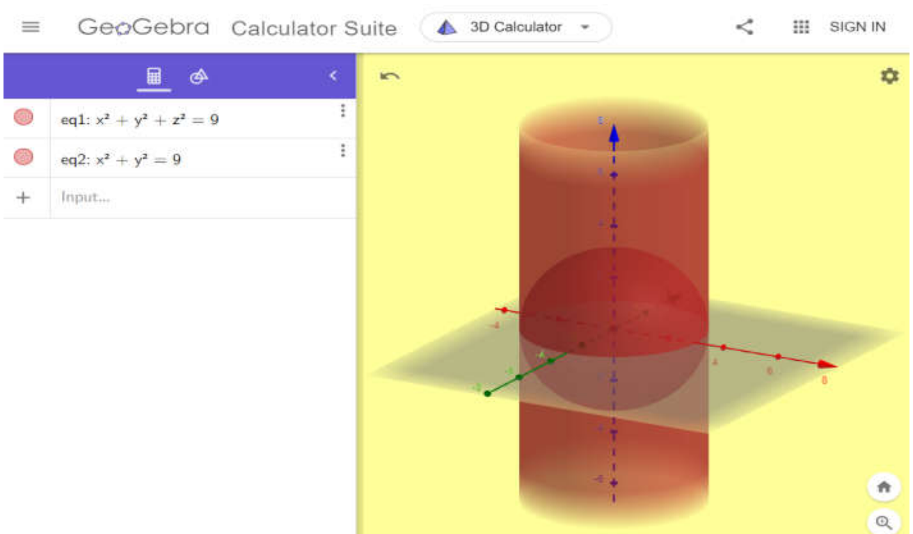
4) Change the background color

Click setting → tick background color → choose color → OK

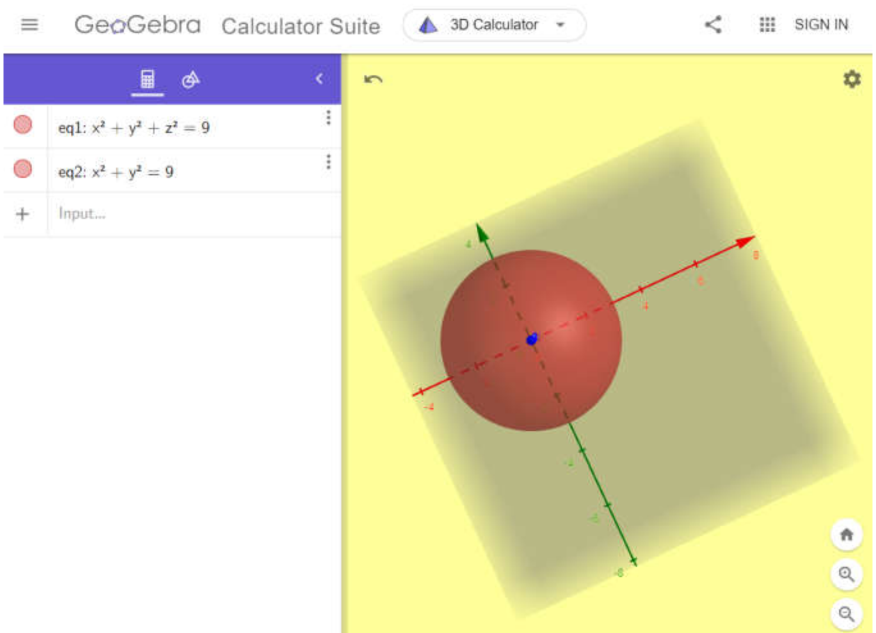





GeoGebra Calculator Suite
3D Calculator
SIGN IN
eq1: $x^2 + y^2 + z^2 = 9$
eq2: $x^2 + y^2 = 9$
Input...
Choose a color
Recent
Other
Preview
White 255, 255, 255 (#FFFFFF)
OK
CANCEL
RESET

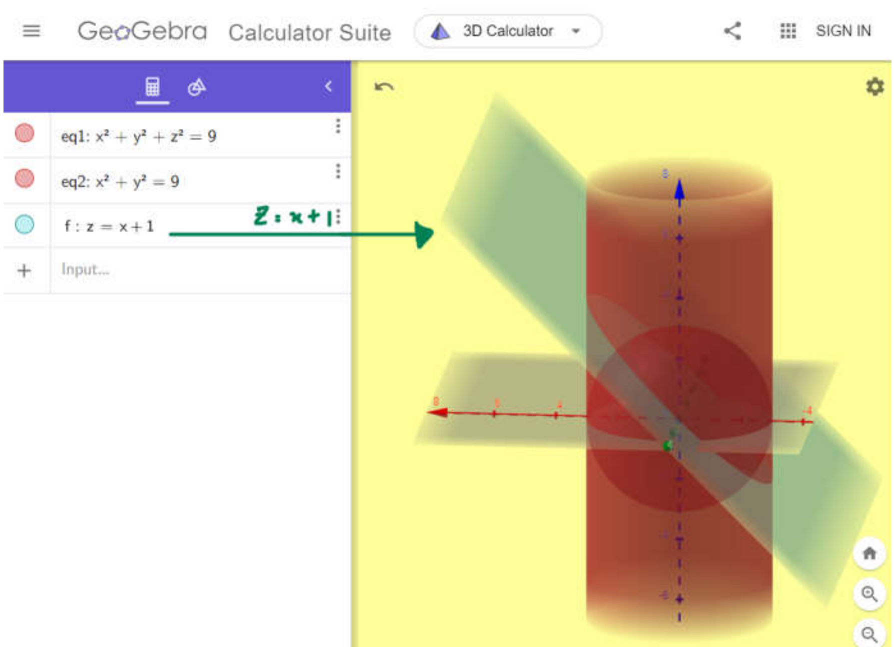

GeoGebra Calculator Suite
3D Calculator
SIGN IN
eq1: $x^2 + y^2 + z^2 = 9$
eq2: $x^2 + y^2 = 9$
Input...

5) Rotate/spin the figure
By click and drag the mouse



6) May insert another function

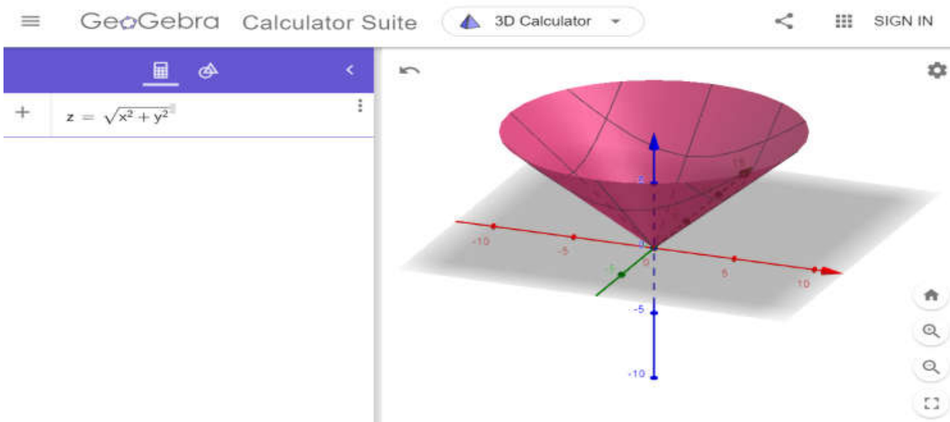
$z = x + 1$



Example 3

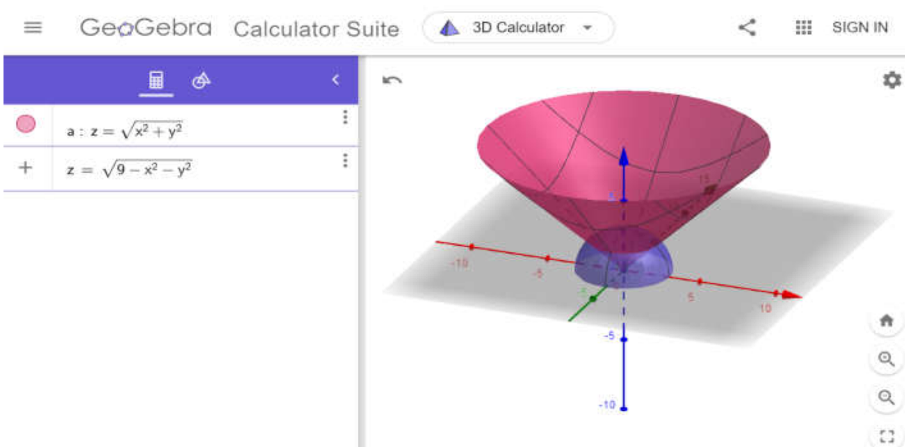
1) Insert the first function

$$z = \sqrt{x^2 + y^2}$$

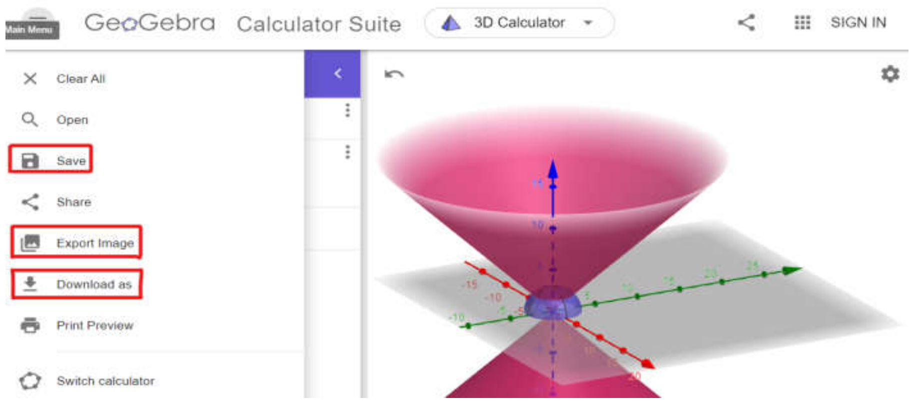


2) Insert the second function

$$z = \sqrt{9 - x^2 - y^2}$$



3) **Save/export/download the graph**
   Click main menu → Save or Export or Download



EXERCISE

Sketch the following graph and state the intersection point(s) if any.

Question 1

$$y = x^2(x-2)$$

$$y = x(6-x)$$

Question 2

$$y = \frac{3}{x}$$

$$y = 2x + 5$$

Question 3

$$x = y^2$$

$$x = y + 2$$

Question 4

$$y = \sqrt{25 - x^2}$$

$$y = 3$$

EXERCISE

Sketch the following graph.

Question 1

$$z = \sqrt{4 - x^2 - y^2}$$

$$x^2 + y^2 = 4$$

$$z = 0$$

Question 2

$$z = 1$$

$$x^2 + y^2 = 9$$

$$x + z = 5$$

Question 3

$$x^2 + y^2 + z^2 = 16$$

$$z = \sqrt{x^2 + y^2}$$

Question 4

$$x^2 + y^2 + z^2 = 9$$

$$x^2 + y^2 = 4$$

FES455

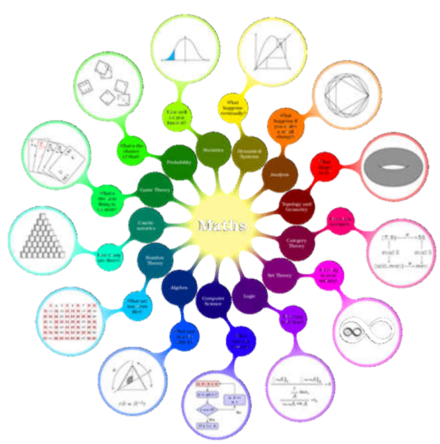
Module 4

Matrix and Vectors

Presented by

Siti Asmah Mohamed

Rafizah Kechil



MODULE 4

MATRIX'S DETERMINANT, DOT PRODUCT AND CROSS PRODUCT

PREPARED BY
RAFIZAH BINTI KECHIL, SITI ASMAH BINTI MOHAMED

INTRODUCTION

A matrix's determinant is a scalar value that is a function of the square matrix's entries. A matrix's determinant is denoted by det(A), det A, or |A|.

The dot product, also known as the scalar product, is an algebraic operation that returns a single number from two equal-length sequences of numbers. The dot product, denoted by the symbol ·, is the sum of the products of the corresponding entries of the two sequences of numbers in a vector.

The cross product, also known as the vector product, is a three-dimensional operation on two vectors denoted by the symbol X. A determinant of a special 3X3 matrix can be used to express the cross product.

In this module, students will learn how to calculate the matrix's determinant, the dot product of a vector and the cross product of a vector.

DETERMINANT

The determinant of a square matrix A is a real number denoted by det(A) or |A|.

2 x 2 determinant:

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

3 x 3 determinant:

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

$$= aei - afh - bdi + bfg + cdh - ceg$$

$$= aei + bfg + cdh - ceg - bdi - afh$$

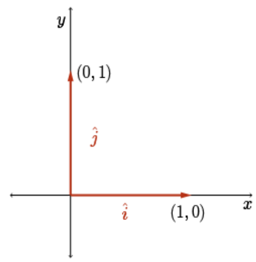
Each determinant of a 2 × 2 matrix in this equation is called a minor of the matrix A. This procedure can be extended to give a recursive definition for the determinant of an n × n matrix, known as Laplace expansion.

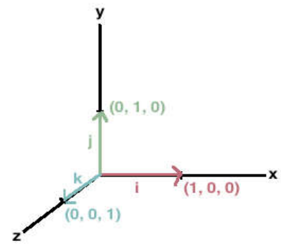
VECTORS

Geometrically, a vector can be visualized as a directed line segment or arrow in 2 and 3 dimensions. In order to do calculations involving vectors, it is necessary to introduce coordinate axes. Similar to a point in the plane and space, a vector can be represented by a list of numbers. The list of numbers representing a point is called coordinates while in vectors it is termed as components. Hence, a vector is specified by giving its components directions.

The coordinate axes are specified by unit vectors that lie along each of the axes.

The components of unit vectors (plane) are defined by $\vec{i}=<1,0>$, $\vec{j}=<0,1>$	The components of unit vectors (space) are defined by $\vec{i}=<1,0,0>$, $\vec{j}=<0,1,0>$, $\vec{k}=<0,0,1>$





These vectors are called standard basis vectors and are denoted by $\vec{\imath}$ and $\vec{\jmath}$ (in two dimension) and $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$ (in three dimension) .

DOT PRODUCT (SCALAR PRODUCT)

If $\vec{p} = p_x\vec{\imath} + p_y\vec{\jmath} + p_z\vec{k}$ and $\vec{q} = q_x\vec{\imath} + q_y\vec{\jmath} + q_z\vec{k}$ are two vectors, the dot product denoted $\vec{p} \bullet \vec{q}$ is defined as

$$\vec{p} \bullet \vec{q} = < p_x, p_y, p_z > \bullet < q_x, q_y, q_z >$$

$$= p_x q_x + p_y q_y + p_z q_z$$

The dot product of two vectors is defined in the plane and space. The dot product $\vec{p} \bullet \vec{q}$ of two vectors is a real number (scalar).

Steps Solving: Dot Product

- Identify components of vector $\vec{p}$
- Identify components of vector $\vec{q}$
- Sum the product of the corresponding component

CROSS PRODUCT

The cross product of two vectors is only defined for vectors in space and will produce another vector that is perpendicular to both vectors. The definition of cross product states the relationships between determinant and cross product.

If $\vec{p} = p_x\vec{i} + p_y\vec{j} + p_z\vec{k}$ and $\vec{q} = q_x\vec{i} + q_y\vec{j} + q_z\vec{k}$ are two vectors in space, then cross product denoted $\vec{p} \times \vec{q}$ is the vector defined by

$$\vec{p} \times \vec{q} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ p_x & p_y & p_z \\ q_x & q_y & q_z \end{vmatrix}$$

$$= \begin{vmatrix} p_y & p_z \\ q_y & q_z \end{vmatrix}\vec{i} - \begin{vmatrix} p_x & p_z \\ q_x & q_z \end{vmatrix}\vec{j} + \begin{vmatrix} p_x & p_y \\ q_x & q_y \end{vmatrix}\vec{k}$$

$$= (p_y q_z - p_z q_y)\vec{i} - (p_x q_z - p_z q_x)\vec{j} + (p_x q_y - p_y q_x)\vec{k}$$

example

Compute the determinant for the following matrices.

Example 1

$$A = \begin{pmatrix} 2 & -3 \\ 5 & 3 \end{pmatrix}$$

Solution :

Instruction		Input
Step 1: Use the notation determinant.	i.	$\det(A) = \begin{vmatrix} 2 & -3 \\ 5 & 3 \end{vmatrix}$
Step 2: Apply the following formula: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$	ii.	$\begin{vmatrix} 2 & -3 \\ 5 & 3 \end{vmatrix} = 2x3 - (-3)x5$
Step 3: Sum up.	iii.	$2x3 - (-3)x5 = 6 + 15$ $= 21$

Example 2

$$A = \begin{pmatrix} -3 & 2 & 2 \\ -1 & 5 & 3 \\ 2 & 4 & -3 \end{pmatrix}$$

Solution:

Instruction		**Input**
Step 1: Use the notation determinant.	i.	$\det(A) = \begin{vmatrix} -3 & 2 & 2 \\ -1 & 5 & 3 \\ 2 & 4 & -3 \end{vmatrix}$
Step 2: Reduce a 3X3 determinant to a 2X2 determinant by using the following formula: $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c$	ii.	$\begin{vmatrix} -3 & 2 & 2 \\ -1 & 5 & 3 \\ 2 & 4 & -3 \end{vmatrix} = -3\begin{vmatrix} 5 & 3 \\ 4 & -3 \end{vmatrix} - 2\begin{vmatrix} -1 & 3 \\ 2 & -3 \end{vmatrix} + 2\begin{vmatrix} -1 & 5 \\ 2 & 4 \end{vmatrix}$
Step 3: Find a 2x2 determinant by using the following formula: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$	iii.	$-3\begin{vmatrix} 5 & 3 \\ 4 & -3 \end{vmatrix} - 2\begin{vmatrix} -1 & 3 \\ 2 & -3 \end{vmatrix} + 2\begin{vmatrix} -1 & 5 \\ 2 & 4 \end{vmatrix}$ $= -3((5)(-3) - (3)(4)) - 2((-1)(-3) - (3)(2)) + 2((-1)(4)$ $= -3(-15 - 12) - 2(3 - 6) + 2(-4 - 10)$ $= -3(-37) - 2(-3) + 2(-14)$ $= 111 + 6 - 28$
Step 4: Sum up.	iv.	$111 + 6 - 28 = 89$

DOT PRODUCT

example

Example 1

If $\vec{a} = -2\vec{\imath} + 6\vec{\jmath}$ and $\vec{b} = 4\vec{\imath} + 9\vec{\jmath} + \vec{k}$, evaluate

i. $\vec{a} \bullet \vec{a}$

ii. $\vec{a} \bullet \vec{b}$

iii. $\vec{b} \bullet \vec{b}$

Solution:

Instruction	**Input**
Step 1: Write the vectors in component form.	i. $\vec{a}$=<-2, 6, 0 > and $\vec{a}$=<-2, 6, 0 >
Step 2: Apply the formula for dot product by finding the product for each corresponding component.	$\vec{a} \bullet \vec{a}$ =<-2, 6, 0 > • <-2, 6, 0 > =(-2)(-2)+(6)(6)+(0)(0)
Step3: Compute the sum.	$\vec{a} \bullet \vec{a}$ =4 + 36+0 =40

Instruction	**Input**
Step 1: Write the vectors in component form.	ii. $\vec{a}$=< −2, 6, o> and $\vec{b}$=<4, 9, 1>
Step 2: Apply the formula for dot product by finding the product for each corresponding component.	$\vec{a} \bullet \vec{b}$ =<-2, 6, 0 > • <4, 9, 1 > =(-2)(4)+(6)(9)+(0)(1)

Step3: Compute the sum.	$\vec{a} \bullet \vec{b} = -8 + 54 + 0$ $= 46$

Instruction	Input
Step 1: Write the vectors in component form.	iii. $\vec{b} = <4, 9, 1>$ and $\vec{b} = <4, 9, 1>$
Step 2: Apply the formula for dot product by finding the product for each corresponding component.	$\vec{b} \bullet \vec{b} = <4, 9, 1> \bullet <4, 9, 1>$ $=(4)(4)+(9)(9)+(1)(1)$
Step 3: Compute the sum.	$\vec{b} \bullet \vec{b} = 16 + 81 + 1$ $= 98$

example

Example 1

If $\vec{a} = -2\vec{\imath} + 6\vec{\jmath}$ and $\vec{b} = 4\vec{\imath} + 2\vec{\jmath} - \vec{k}$, evaluate

i. $\vec{a}\mathrm{x}\vec{b}$

ii. $\vec{b}\mathrm{x}\vec{a}$

Solution:

Instruction	Input
Step 1: Write the vectors in component form.	i. $\vec{a}$=<-2, 6, $0>$ and $\vec{b}$=<4, 2, $-1>$
Step 2: Form a third order determinant and place the unit vectors and component on right place rows.	$\vec{a}\,\mathrm{x}\,\vec{b} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ -2 & 6 & 0 \\ 4 & 2 & -1 \end{vmatrix}$
Step 3: Compute the determinant.	$\vec{a}\,\mathrm{x}\,\vec{b} = \begin{vmatrix} 6 & 0 \\ 2 & -1 \end{vmatrix}\vec{\imath} - \begin{vmatrix} -2 & 0 \\ 4 & -1 \end{vmatrix}\vec{\jmath} + \begin{vmatrix} -2 & 6 \\ 4 & 2 \end{vmatrix}\vec{k}$

$$= (-6 - 0)\vec{\imath} - (2 - 0)\vec{\jmath} + (-4 - 24)\vec{k}$$

$$= -6\vec{\imath} - 2\vec{\jmath} - 28\vec{k}$$

$$=< -6, -2, -28>$$

Instruction	Input
Step 1: Write the vectors in component form.	ii. $\vec{b}=< 4, 2, -1 >, \ \vec{a} =< -2, 6, 0>$
Step 2: Form a third order determinant and place the unit vectors and component on right place rows.	$\vec{b} \times \vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 2 & -1 \\ -2 & 6 & 0 \end{vmatrix}$
Step 3: Compute the determinant.	$\vec{b} \times \vec{a} = \begin{vmatrix} 2 & -1 \\ 6 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 4 & -1 \\ -2 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 4 & 2 \\ -2 & 6 \end{vmatrix} \vec{k}$

$$= (0 - (-6))\vec{i} - (0 - 2)\vec{j} + (24 - (-4))\vec{k}$$

$$= 6\vec{i} + 2\vec{j} + 28\vec{k}$$

$$=< 6, 2, 28>$$

EXERCISE

Find the determinant for the following matrices.

Question 1

$$A = \begin{pmatrix} 3 & 2 \\ 7 & -2 \end{pmatrix}$$

Question 2

$$A = \begin{pmatrix} 13 & -4 \\ 4 & -3 \end{pmatrix}$$

Question 3

$$A = \begin{pmatrix} 1 & 4 & 6 \\ -4 & 2 & -1 \\ 6 & -2 & 5 \end{pmatrix}$$

Question 4

$$A = \begin{pmatrix} 11 & 0 & 3 \\ -2 & 6 & -2 \\ 3 & 1 & 10 \end{pmatrix}$$

Answer:

1. -20
2. -23
3. 40
4. 622

EXERCISE

Compute the dot product $\vec{a} \bullet \vec{b}$ where

Question 1

$$\vec{a} = 3\vec{i} + 4\vec{j} \quad \text{and} \quad \vec{b} = 2\vec{i} - 3\vec{j}$$

Question 2

$$\vec{a} = 2\vec{i} - 3\vec{j} + 4\vec{k} \text{ and } \vec{b} = 6\vec{j} + 5\vec{k}$$

Question 3

$$\vec{a} = 2\vec{i} - \vec{j} + \vec{k} \text{ and } \vec{b} = \vec{i} - \vec{k}$$

Question 4

$$\vec{a} = 5\vec{i} - 7\vec{j} + \vec{k} \text{ and } \vec{b} = 2\vec{i} - \vec{j} + 3\vec{k}$$

Answer:

1. -6
2. -2
3. 1
4. 20

EXERCISE

Compute the cross product $\vec{a}\mathrm{x}\vec{b}$ where

Question 1

$$\vec{a} = 2\vec{\imath} + \vec{\jmath} \quad \text{and} \quad \vec{b} = -\vec{\imath} + \vec{\jmath}$$

Question 2

$$\vec{a} = -\vec{\imath} + 3\vec{\jmath} + \vec{k} \ \text{ and } \ \vec{b} = 4\vec{\imath} + \vec{\jmath} + 2\vec{k}$$

Question 3

$$\vec{a} = 3\vec{\imath} + \vec{\jmath} - 4\vec{k} \ \text{ and } \ \vec{b} = 2\vec{\imath} + \vec{\jmath} - 3\vec{k}$$

Question 4

$$\vec{a} = \vec{\imath} - \vec{\jmath} + \vec{k} \ \text{ and } \ \vec{b} = 2\vec{\imath} - 5\vec{\jmath} - 3\vec{k}$$

Answer:

1. $< 0, 0, 3 >$
2. $< 5, 6, -13 >$
3. $< 1, 1, 1 >$
4. $< 8,\ 5, -3 >$

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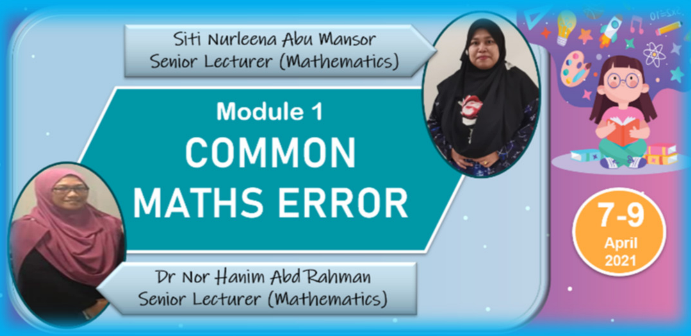

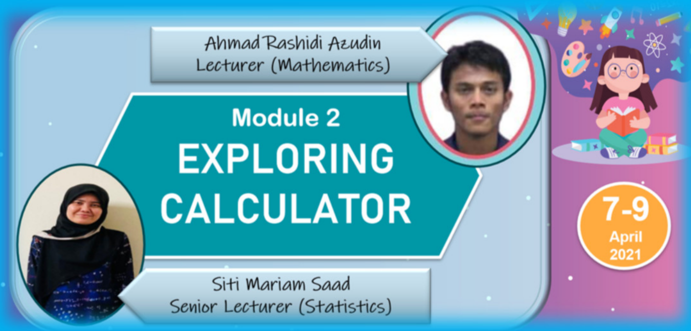

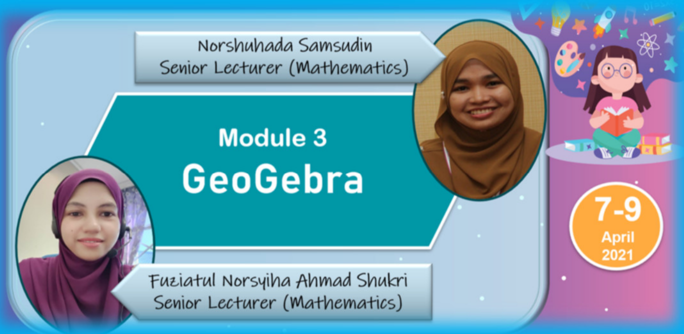

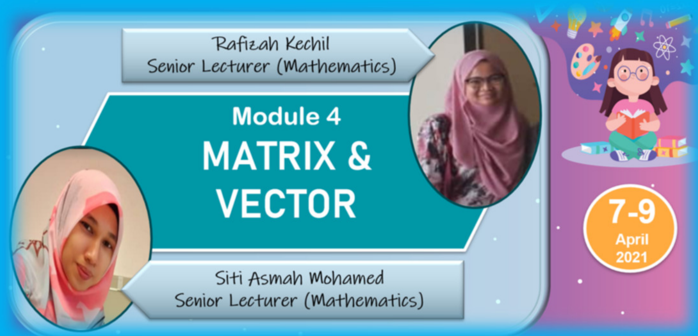

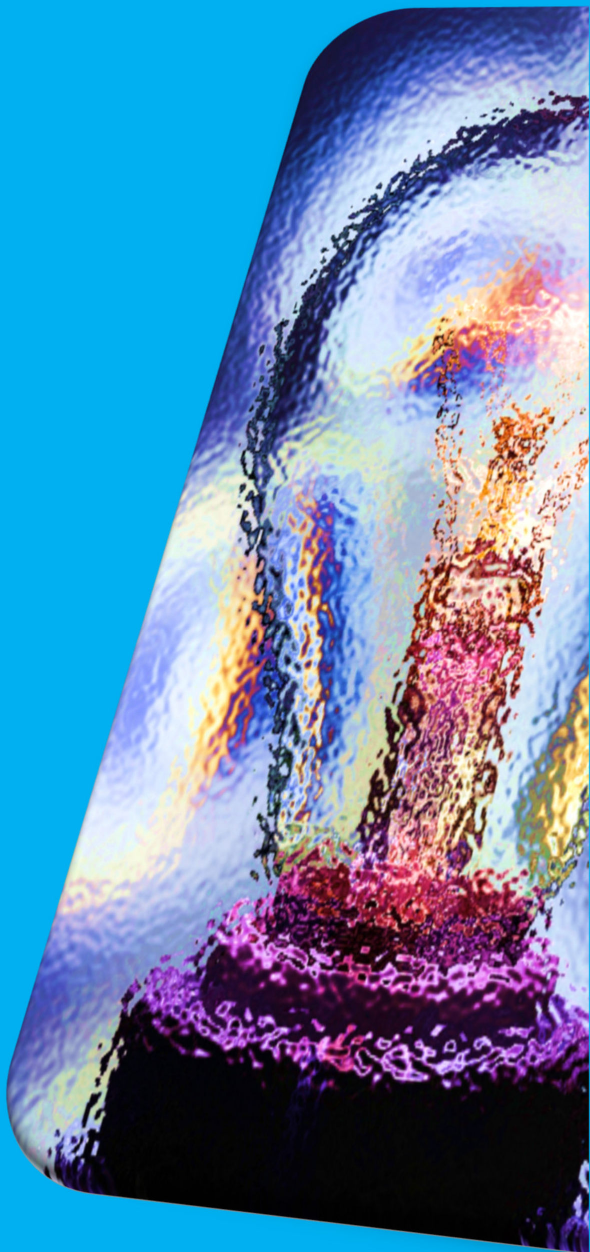

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